Numerics for ultra-intense fields

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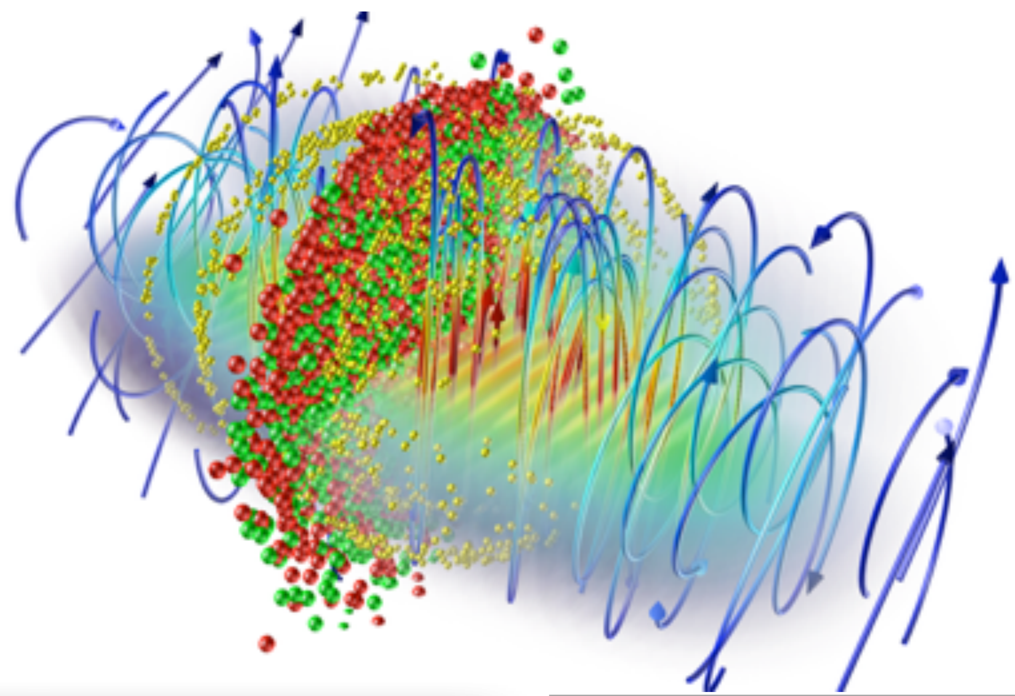

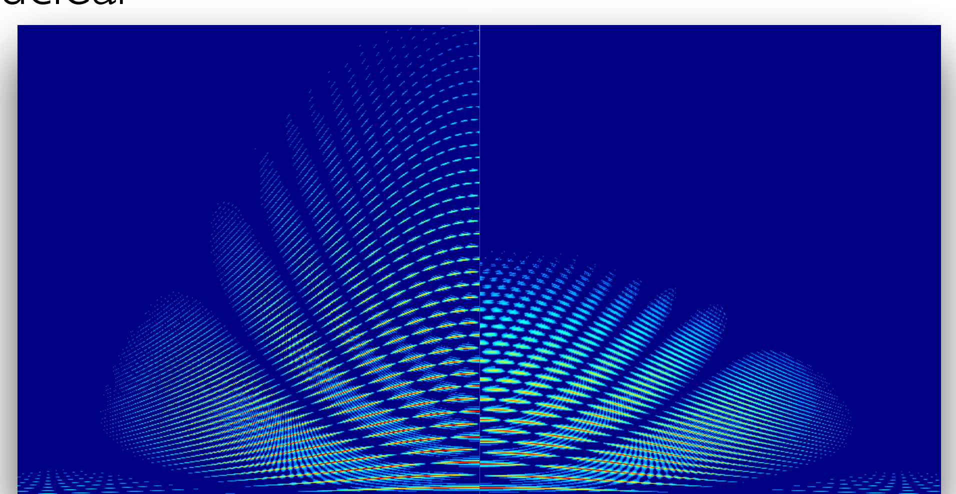

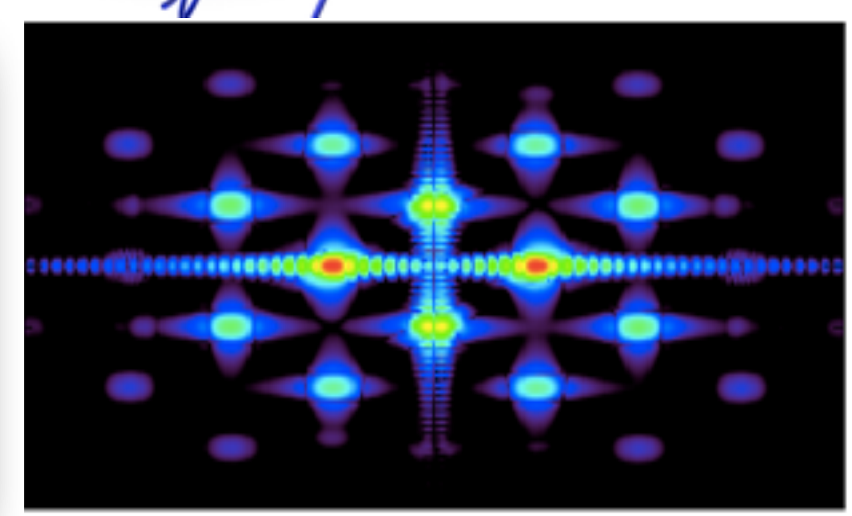

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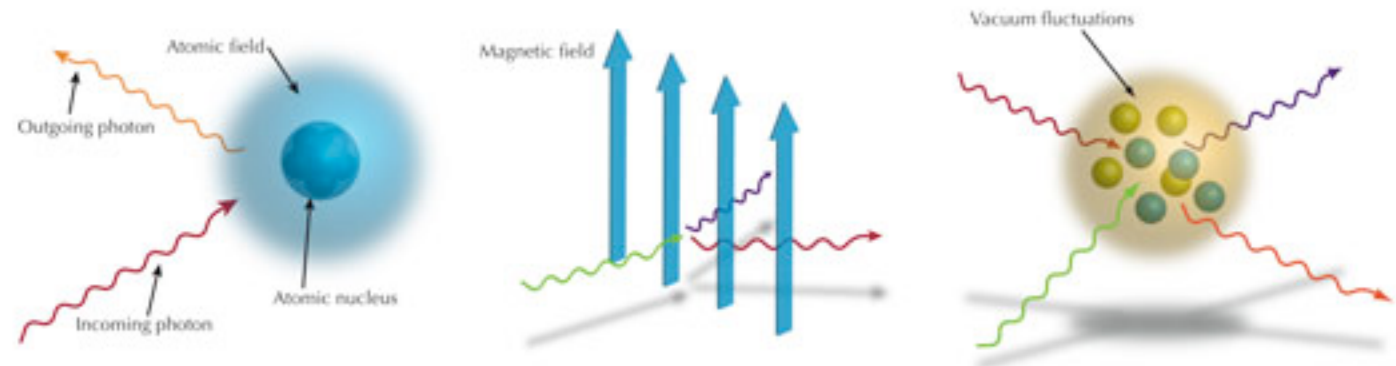
Multi Petawatt Lasers

Modern laser pulses

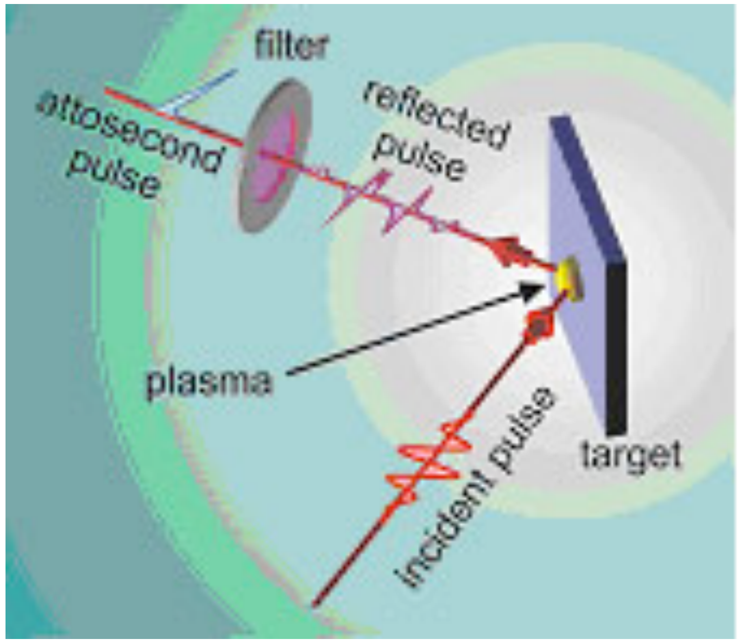
- Pulse duration : 30-100 fs
- Focal width ~ μm
- Intensity ~10^{21} - 10^{25} W/cm^2
- Extreme acceleration regime



QED Photons interaction

Possibility of probing new physics

- Secondary sources : particles and radiation
- High fields : quantum vacuum and quantum dynamics
- Atto-science : generation of flashes of light
- Radiography : phase imaging for medical technique
- Hadron therapy : cancer treatment

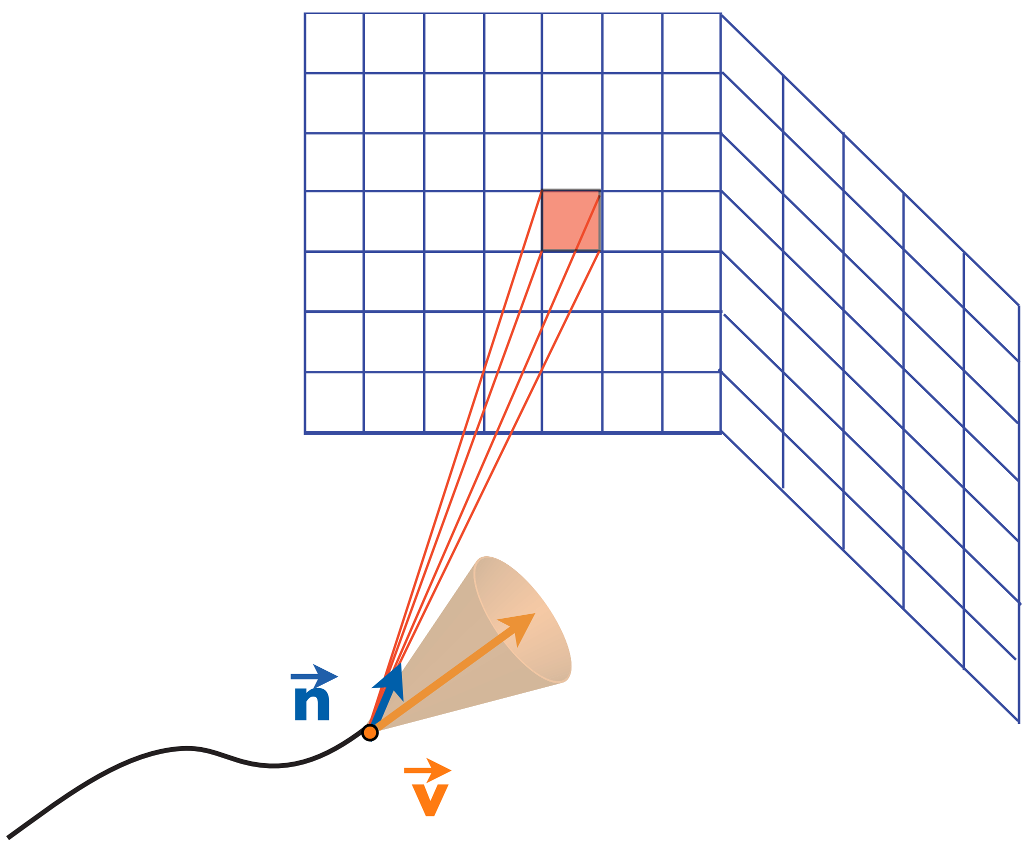


ELI

High power atto-second pulse generation

Contents

Post-processing radiation from PIC trajectories



Spectrum

trajectory without radiation damping

+

general

$$\frac{d^2I(\omega)}{d\omega d\Omega} = \frac{e^2}{4\pi c}\left|\sum_p \int_{-\infty}^{+\infty} \frac{\mathbf{n}\times[(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}]}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^2}\exp\left[i\omega\left(t+R/c\right)\right]dt\right|^2$$

far-field

$$\frac{d^2I(\omega)}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c^3}\left|\sum_p \int_{-\infty}^{+\infty} \mathbf{n}\times(\mathbf{n}\times\boldsymbol{\beta})\exp\left[i\omega\left(t-\mathbf{n}\cdot\mathbf{r}/c\right)\right]dt\right|^2$$

Jackson, J.D., Classical Electrodynamics

Radiated energy

$$E_{pixel} = \frac{e^2}{4\pi c}\sum_p \int \frac{|\mathbf{n}\times[(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}]|^2}{(1-\mathbf{n}\cdot\boldsymbol{\beta})^5 R^2} S_{pixel}\, dt$$

Jackson, J.D., Classical Electrodynamics

Spectrum for classical damping regime

trajectory with radiation damping

+

$$^* \quad \frac{d^2I(\omega)}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c^3}\left|\sum_p \int_{-\infty}^{+\infty} \frac{\mathbf{n}\times(\mathbf{n}\times\boldsymbol{\beta})}{\sqrt{\eta}}\exp\left[i\frac{\omega}{\eta}\left(t-\mathbf{n}\cdot\mathbf{r}/c\right)\right]dt\right|^2$$

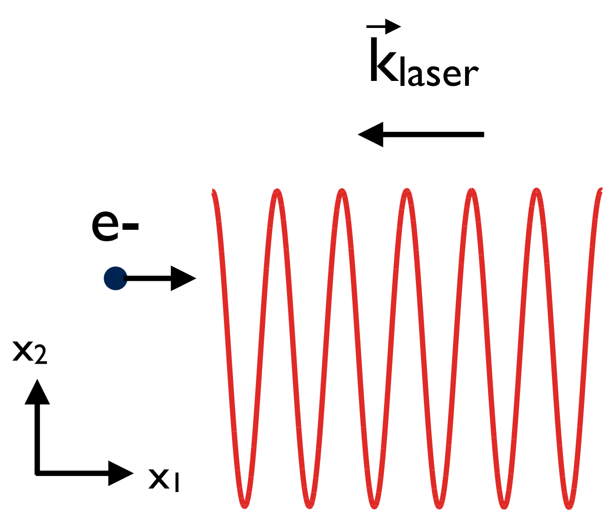
$$\eta = \eta(\omega,\Omega) \simeq 1 - \frac{\hbar\omega'}{mc^2}$$

* Sokolov, et al, Phys. Rev. E 81, 036412 (**2010**)

Derivation from FWW method: J.L. Martins et al, PPCF, 58, 014035 (**2016**)

How does the spectrum change ?

Setup



$a_0 = 2$

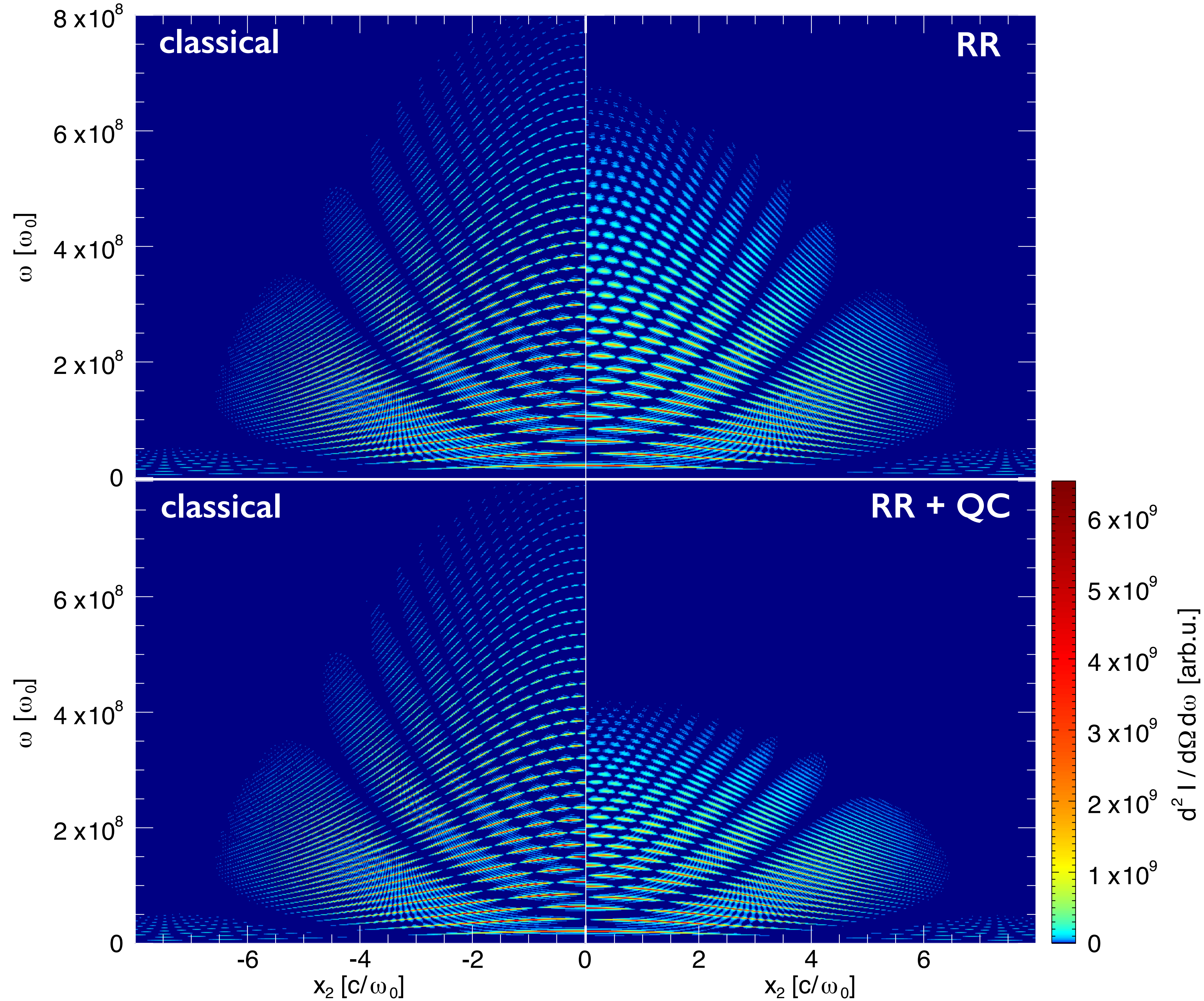
$\lambda = 1$ μm

$10\,T_0$

$\gamma = 4000$

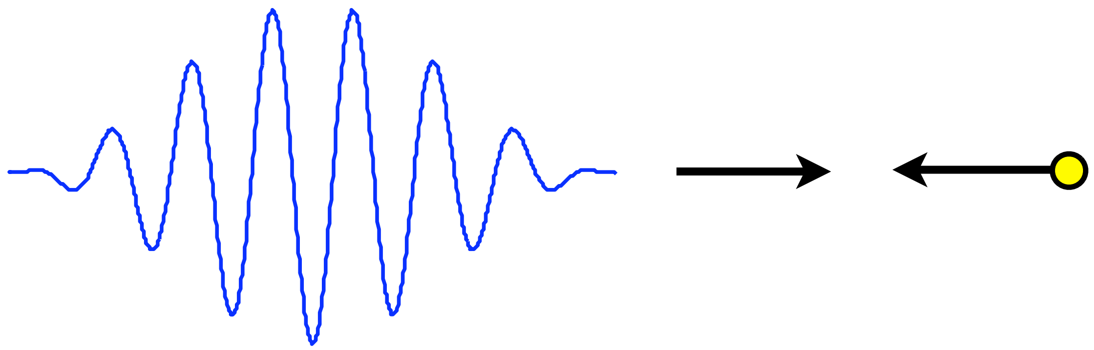
$\Delta E \sim -1\%$

Spectra spatially resolved in polarization direction



Reduced L&L is best for PIC

L&L captures physically relevant solutions of LAD equation*



Without radiation reaction

$$\frac{d\mathbf{p}}{dt} = e\left(\mathbf{E}+\frac{\mathbf{p}}{\gamma mc}\times\mathbf{B}\right)$$

With radiation reaction

A

$$\frac{d\mathbf{p}}{dt} = e\left(\mathbf{E}+\frac{\mathbf{p}}{\gamma mc}\times\mathbf{B}\right)+\frac{2e^3}{3mc^3}\Bigg\{\boxed{\cancel{\gamma\left(\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{\gamma m}\cdot\nabla\right)\mathbf{E}+\frac{\mathbf{p}}{\gamma mc}\times\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{\gamma m}\cdot\nabla\right)\mathbf{B}\right)}}$$

$$+\frac{e}{mc}\left(\mathbf{E}\times\mathbf{B}+\frac{1}{\gamma mc}\mathbf{B}\times(\mathbf{B}\times\mathbf{p})+\frac{1}{\gamma mc}\mathbf{E}(\mathbf{p}\cdot\mathbf{E})\right)-\boxed{\frac{e\gamma}{m^2c^2}\mathbf{p}\left(\left(\mathbf{E}+\frac{\mathbf{p}}{\gamma mc}\times\mathbf{B}\right)^2-\frac{1}{\gamma^2m^2c^2}(\mathbf{E}\cdot\mathbf{p})^2\right)}\Bigg\}$$

B

L&L reduced**

$$\frac{A}{B}\sim\frac{1}{2\pi\gamma a_0}$$

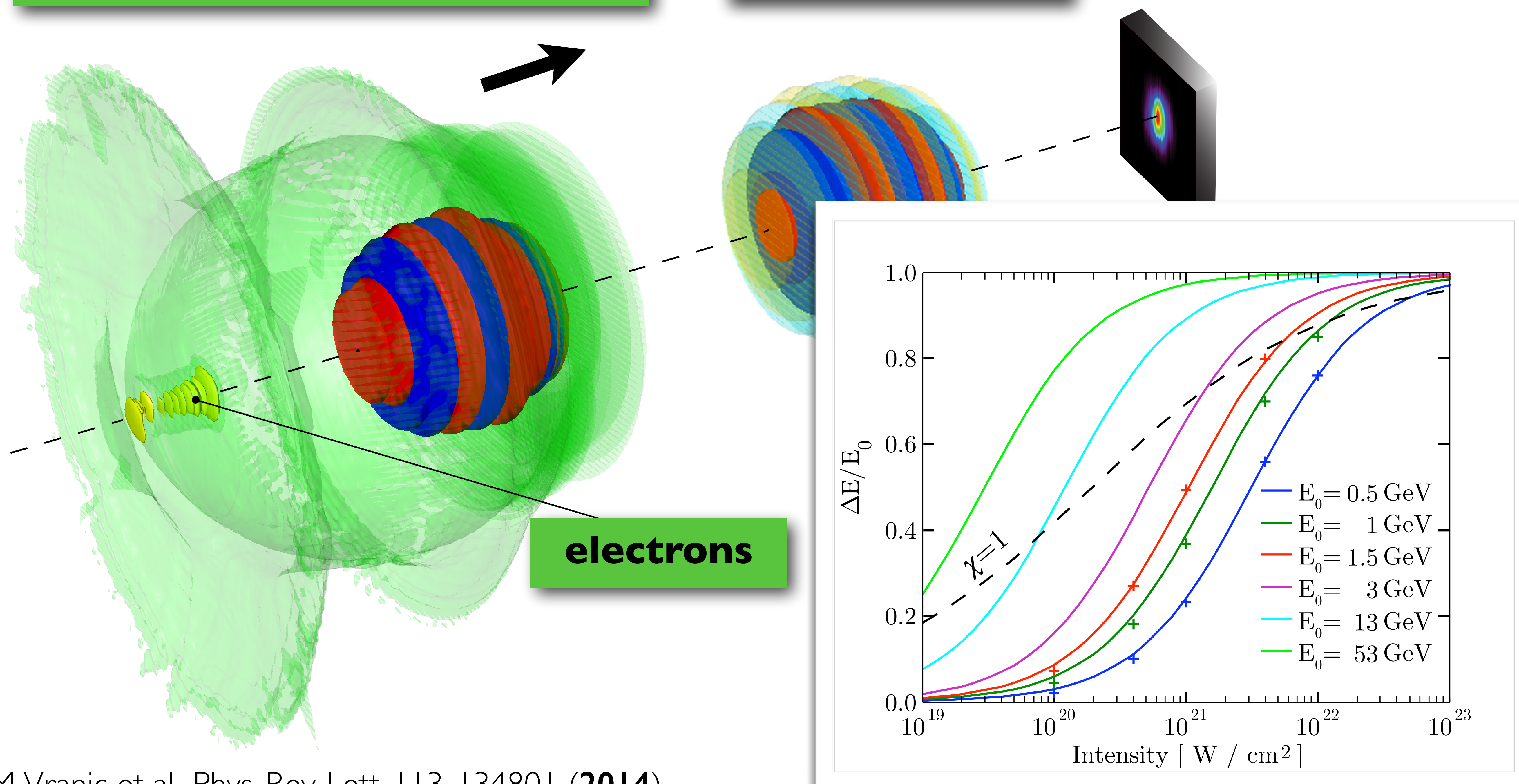
* H. Spohn, Europhys. Lett. 50, 287 - 292 (**2000**)
A. Ilderton and G. Torgrimsson, Phys. Lett. B, 725, 481-486 (**2013**)

All-optical radiation reaction

~ 40% energy loss for a 1 GeV beam at 10^{21} W/cm^2

laser wakefield accelerator in bubble regime

second laser
I ~ 10^{21} W/cm^2



M.Vranic et al., Phys. Rev. Lett. 113, 134801 (**2014**)

Processes in background fields : Loops ...

Vacuum birefringence

Heinsenberg and Euler, Z. Physik **1936**
Karplus and Neuman, Phys. Rev. **1950**

Photon emission/splitting/scattering

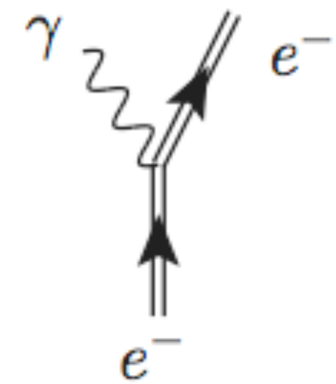
Adler, Annals. Phys. **1971**
Lundstrom et. al, Phys. Rev. **2006**

Schwinger pair production

Schwinger, Phys. Rev. **1951**
Dunne, Gies and Schutzhold, Phys. Rev. **2009**

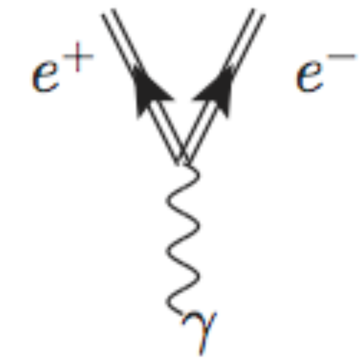
Processes in background fields : Trees ...

Nonlinear Compton scattering



Periodic fields: Nikishov and Ritus **1964**
Pulses: Makenroth and Di Piazza **2011**

Stimulated pair production



Periodic fields: Nikishov and Ritus **1964**
Pulses: Heinzl, Ilderton and Marklund **2010**

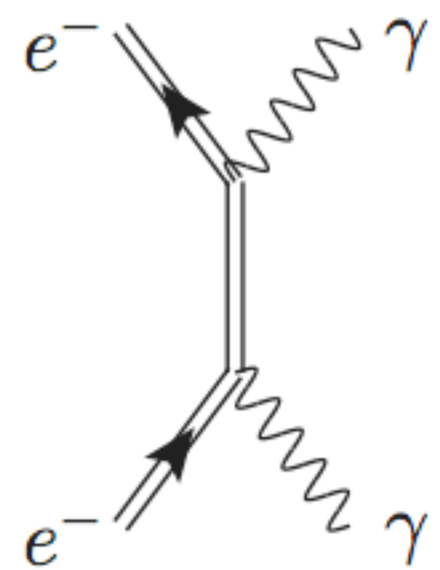
Cascades

Fedotov et al. Phys. Rev. Lett. **2011**
Bell, Kirk et al. Phys. Rev. Lett. **2008**
Elkina et al. Phys. Rev. ST Accel. **2011**

Higher order processes

Strong field Compton

V.P. Oleinik Sov. Phys. JETP **1967**
Ya. B. Zeldonich. Sov. Phys. JETP **1967**

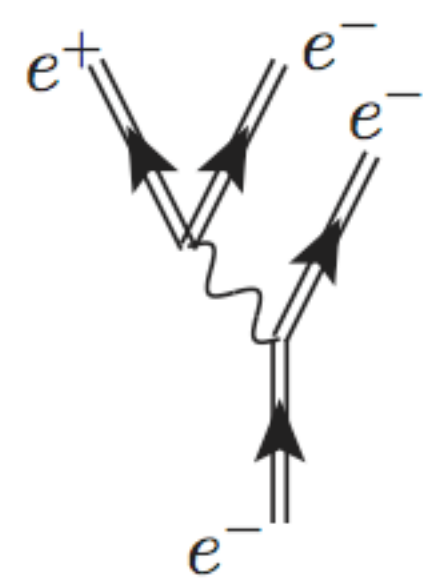


Two-photon pair production

A. Hartin, PhD thesis **2006**

Trident pair production

H. Hu, C. Muller, C. H. Keitel, Phys. Rev. Lett **2010**



Moller scattering

F. Ehlotzky, Rep. Prog. Phys. **2009**

The fundamental χ parameter

- **Schwinger field**

$$E_s = \frac{m^2 c^3}{e\hbar}$$

→ **Pair creation probability :**

$$W \propto \exp(-\pi E_s/E)$$

- **Let us introduce the parameter**

$$\chi = \frac{E}{E_s}$$

- **And generalized in any frame**

$$\chi = \frac{1}{E_s}\sqrt{(\gamma \mathbf{E} + \frac{\mathbf{p}}{mc} \times \mathbf{B})^2 - (\frac{\mathbf{p}}{mc} \cdot \mathbf{E})^2}$$

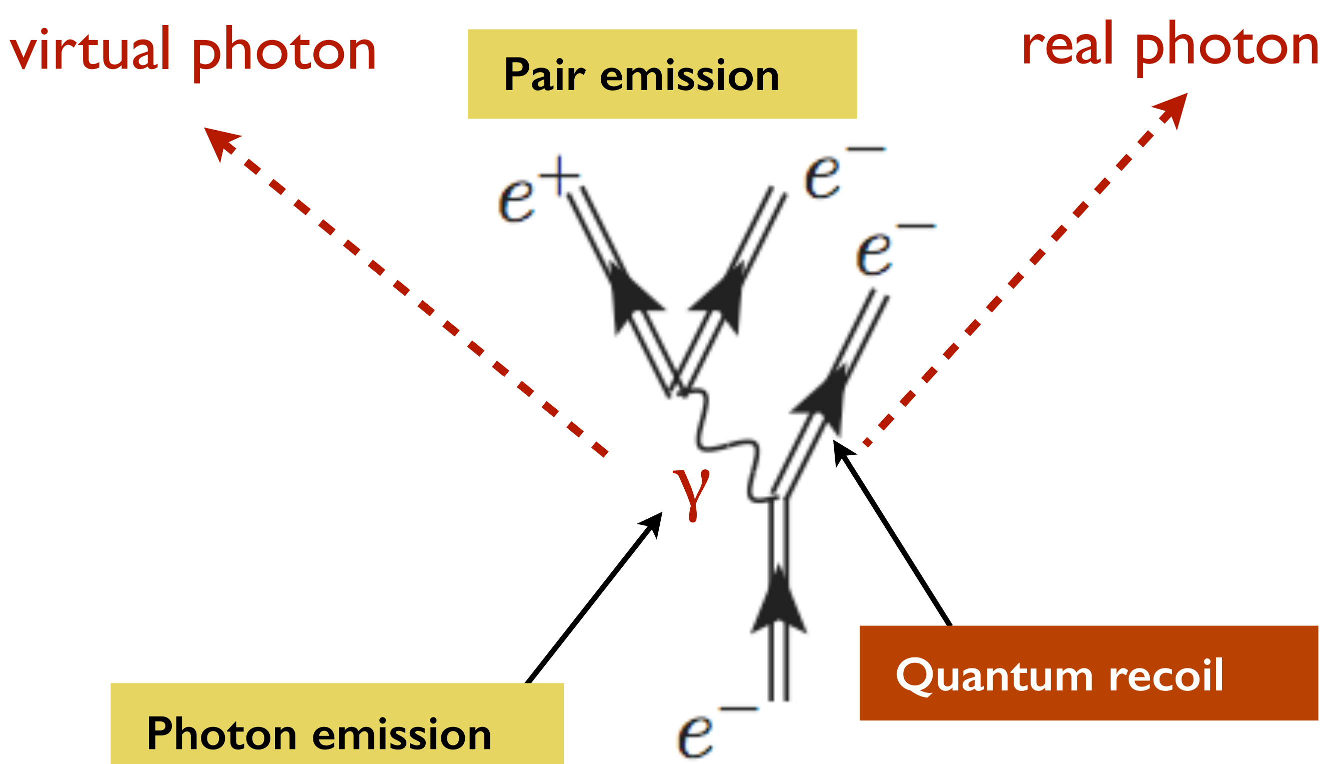
$$\chi \simeq \frac{\gamma E_\perp}{E_s}$$

Other configuration with lower E should allow pair creation ! ←

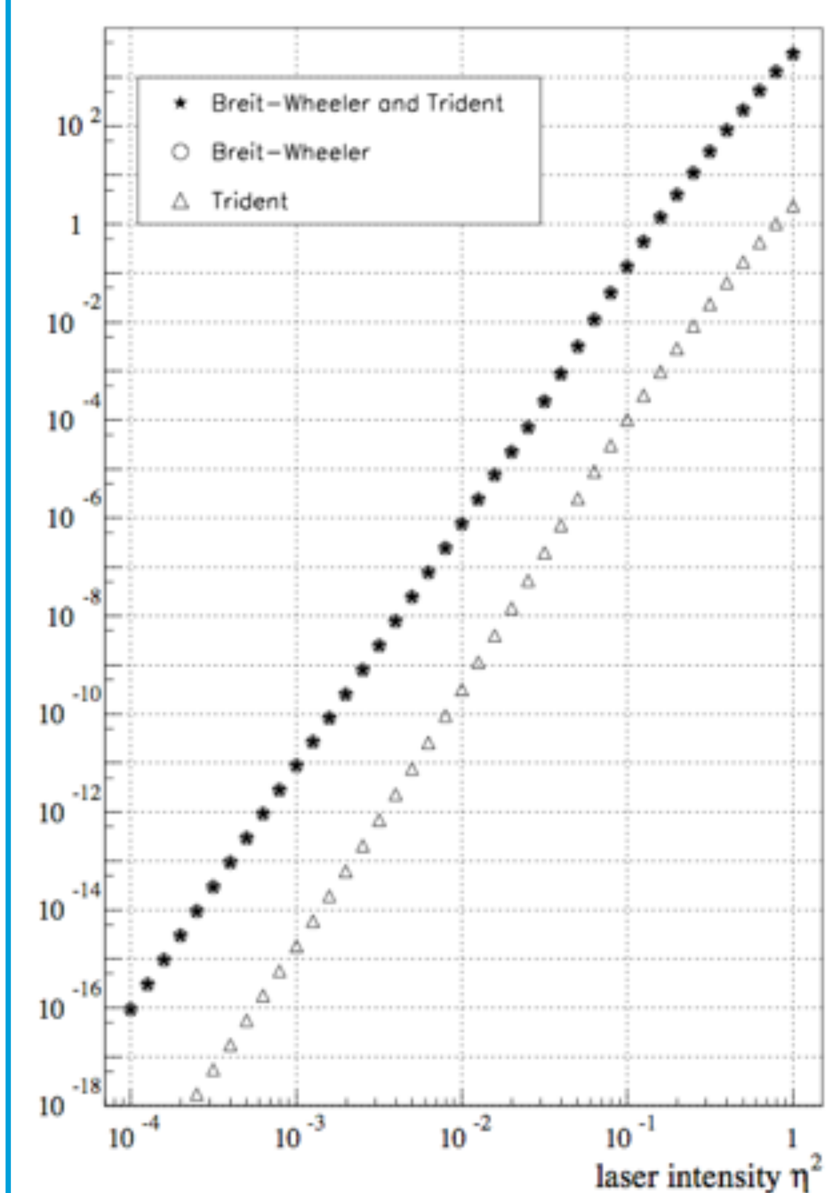
Trident pair production

Two different processes contributing

One step (trident)	Two step (Breit-Wheeler)
$e^- + (\text{laser}) \rightarrow e^- + e^- + e^+$	Non linear Compton scattering: $e^- + (\text{laser}) \rightarrow \gamma + e^-$ Stimulated pair production: $\gamma + (\text{laser}) \rightarrow e^- + e^+$



Rates for the two processes



SLAC E144
Bambers et al. Phys. Rev. D 1999

OSIRIS-QED PIC LOOP

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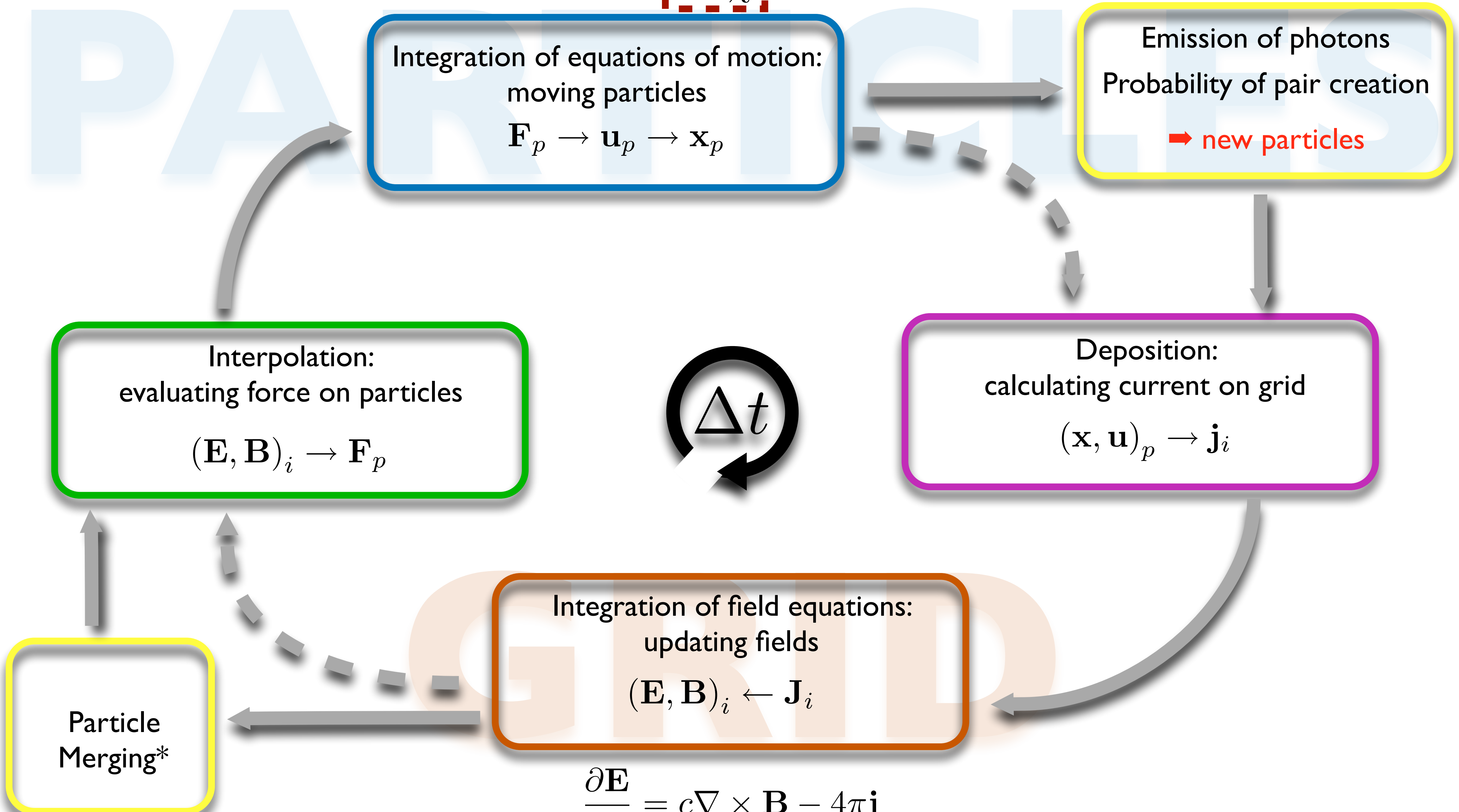
$$\frac{d\mathbf{p}}{dt} = \mathbf{F}_L + \boxed{\frac{d\mathcal{P}_\gamma}{dtd\chi}}$$ Probabilistic



$$\frac{\partial \mathbf{E}}{\partial t} = c\nabla \times \mathbf{B} - 4\pi \mathbf{j}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -c\nabla \times \mathbf{E}$$

E.N Nerush et al., *PRL* 106, 035001 (**2011**)
C. P. Ridgers et al., *PRL,* 108, 165006 (**2012**)
M. Lobet et al., *PRL* 115, 215003 (**2015**)
A. Gonoskov et al., *PRE* 92, 023305 (**2015**) *M. Vranic et al., *CPC* 191, 65-73 (**2015**)

Implementation of QED effects

Radiation Reaction

Different types of Radiation reaction models

$$\frac{d\mathbf{p}}{dt} = \mathbf{F}_L + \begin{cases} \mathbf{F}_{rad} & \text{Continuous damping rate*} \\ \dfrac{d^2P}{dt\,d\chi_\gamma} & \text{QED probabilistic approach**} \end{cases}$$

Implementation in PIC codes

- Continuous damping rate: particle pusher with $\mathbf{F}_{rad}$ $\quad \gamma < 10$

- QED probabilistic approach: particle pusher + Monte Carlo module
  - every Δt : probability of photon emission
  - Select a photon in QED synchrotron spectrum
  - Update particle momentum due to quantum recoil

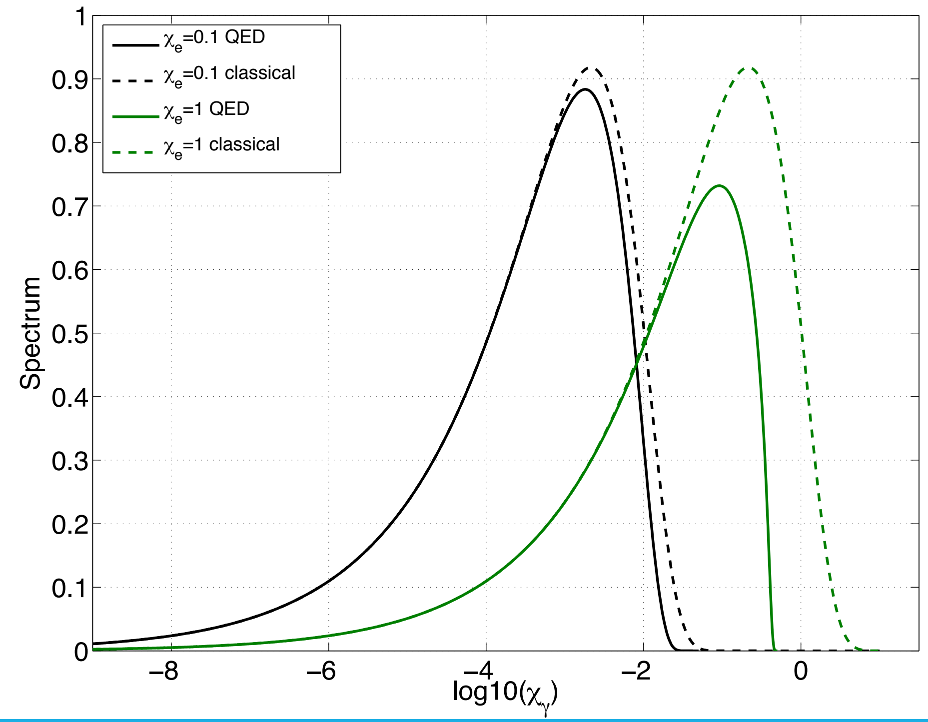
- The QED approach can be generalized to any external EM fields under the conditions: $t_{carac}(\vec{E},\vec{B}) \gg t_{coh} \implies a_0 \gg 1$
  - quasi-static fields
  - weak fields $\quad \chi_e^2 \gg \mathrm{Max}(f,g) \quad (f,g) \ll 1$

$$f = F_{\mu\nu}^2/E_{crit}^2 \qquad g = F_{\mu\nu}^* F_{\mu\nu}/E_{crit}^2 \qquad E_{crit} = m^2c^3/e\hbar \qquad \chi_{e,\gamma} = \frac{|F_{\mu\nu}p^\nu_{e,\gamma}|}{E_{crit}mc}$$

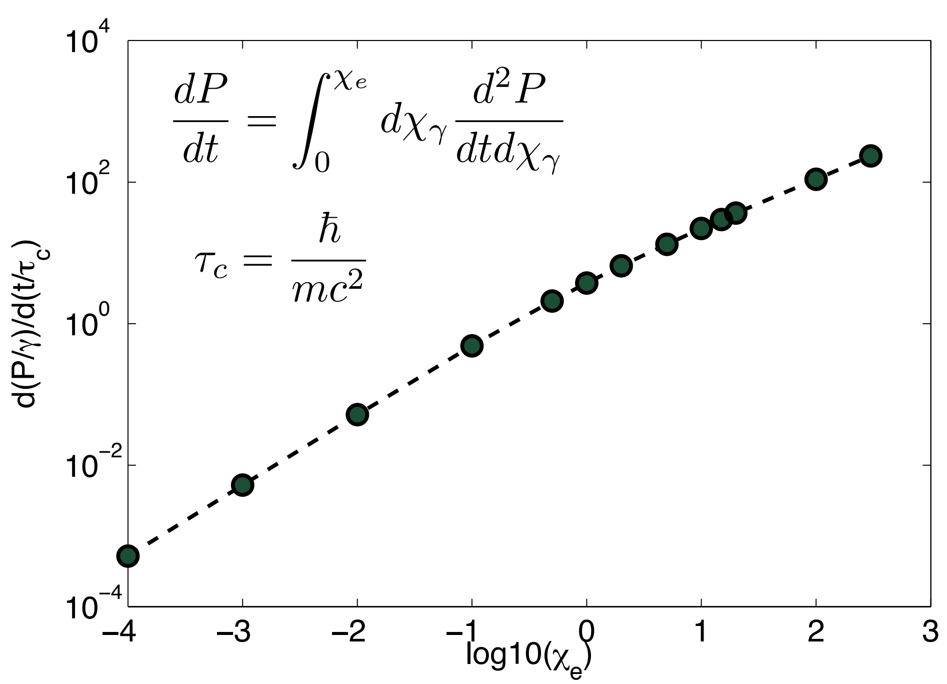
* Landau & Lifshitz (Theory of Fields)

** A.I. Nikishov & V.I. Ritus (1967), N.P. Kelpikov (1954), V.N. Baier & V.M. Katkov (1967)

Synchrotron Spectrum



Emission rate



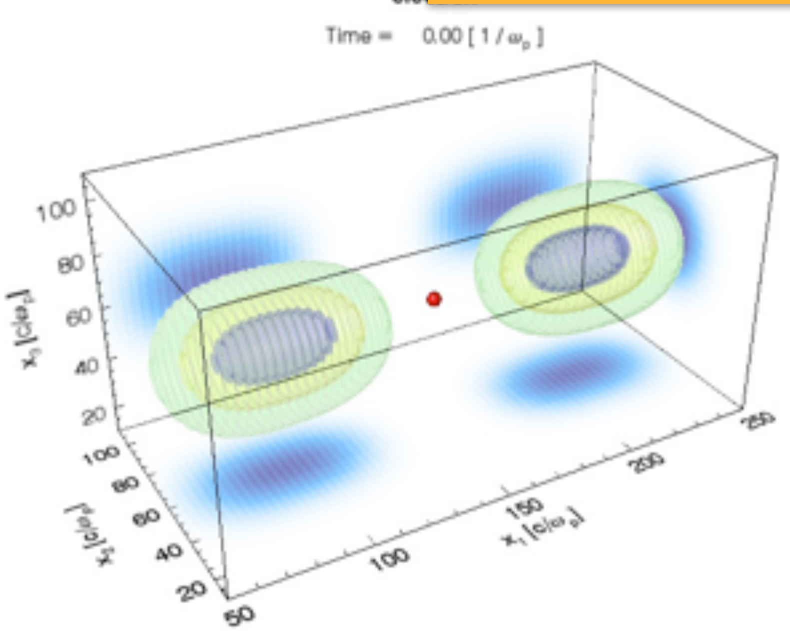
Standing waves configurations

Linear

- electron
- positron

Parameters

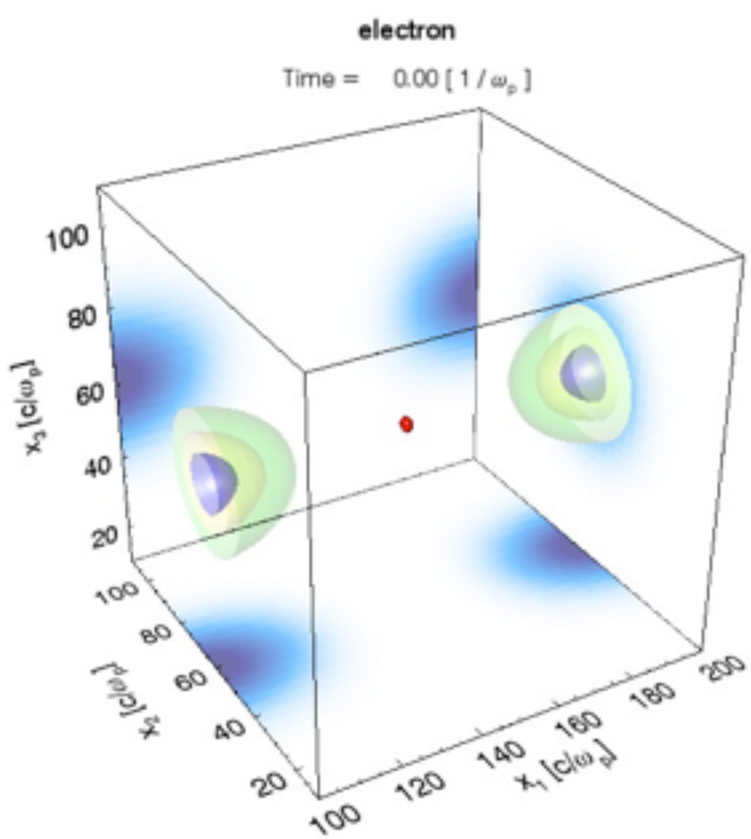
- absorbing boundaries
- a0 = 1000
- λ0 = 1μm
- W0 = 3.2 μm
- τ = 60 fs



Particles remain in the x_1-x_2 plane

Double clockwise

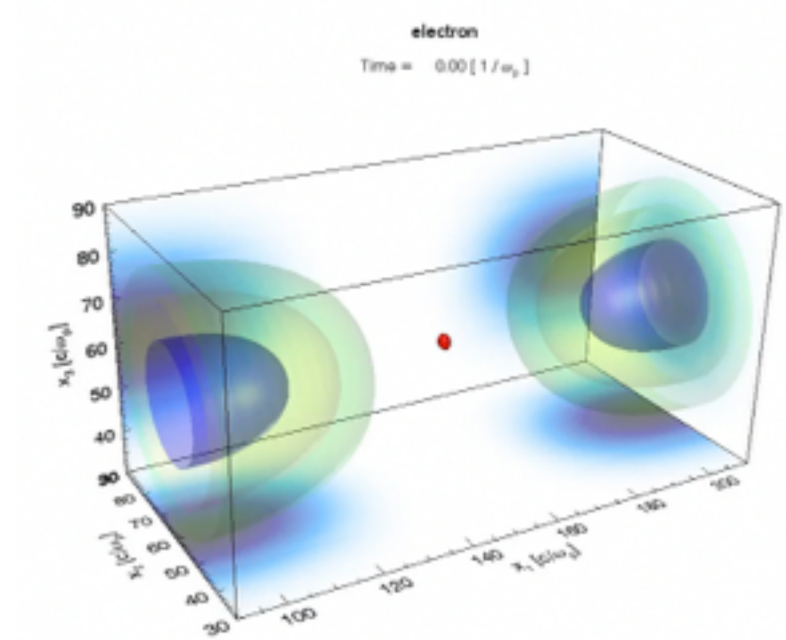
- electron
- positron
- photon



Particles explore the whole space

Clockwise-anti clockwise

- electron
- positron
- photon



Particles rotate mainly in the x_2-x_3 plane

A.R Bell and J. G Kirk PRL, 101, 200403 (**2008**)

Merging algorithm*

Calculate the number of merging cells and their size

Calculate the number of particles within each merging cell

Find the p_{min} and p_{max} of the particles in every merging cell

↓

Bin the momentum space for every merging cell

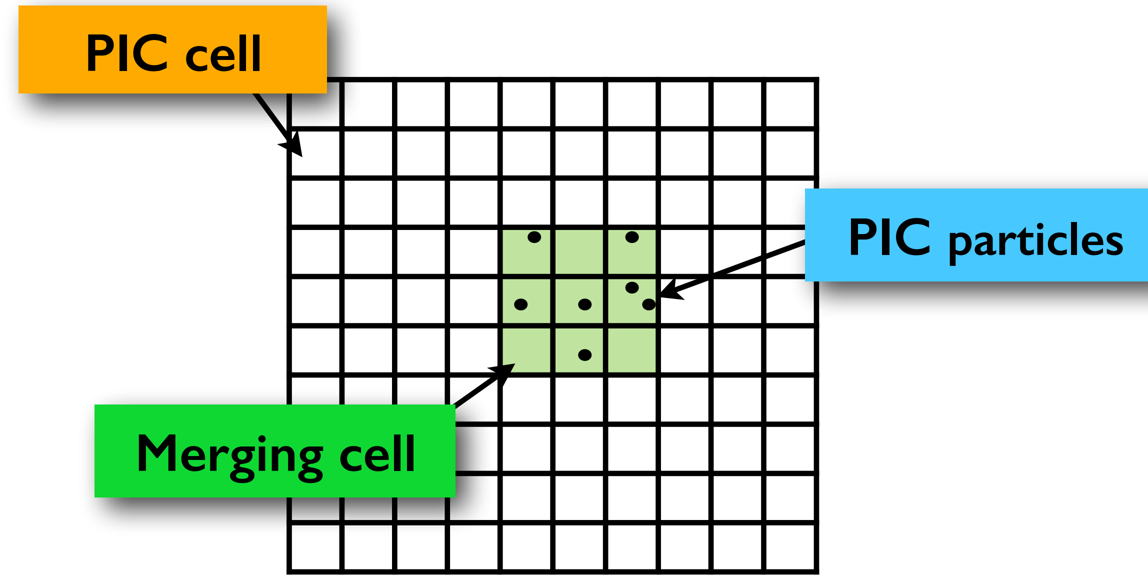
Distribute the particles of every merging cell in its momentum bins

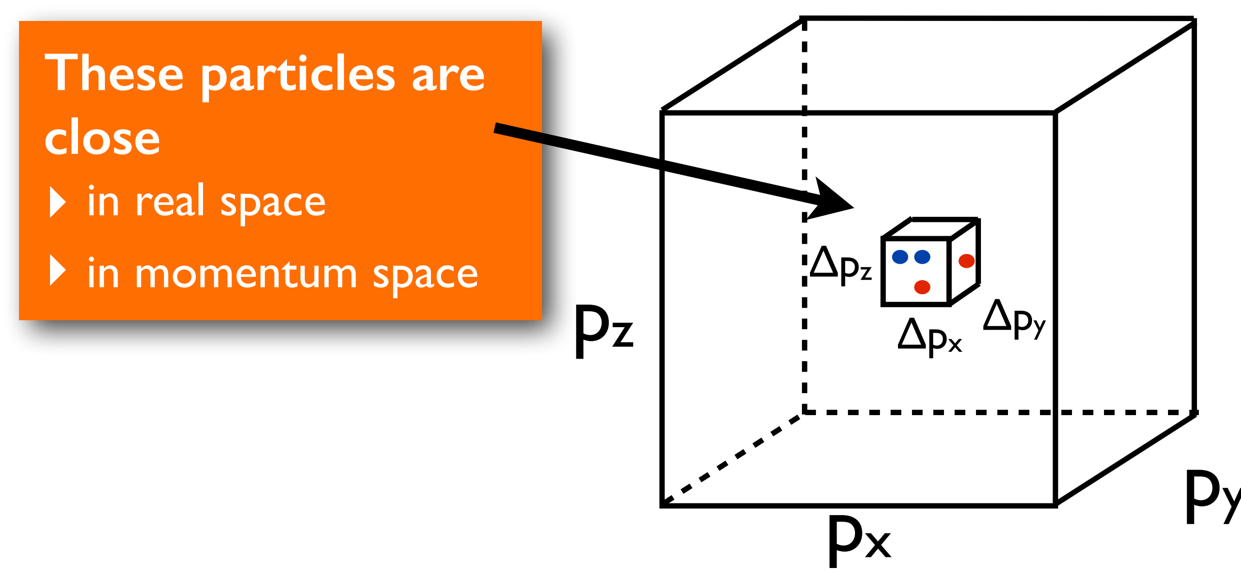
↓

Calculate the total weight, momentum, energy in every momentum bin

Merge the particles in every momentum bin into 2 new particles

Remove all the former particles



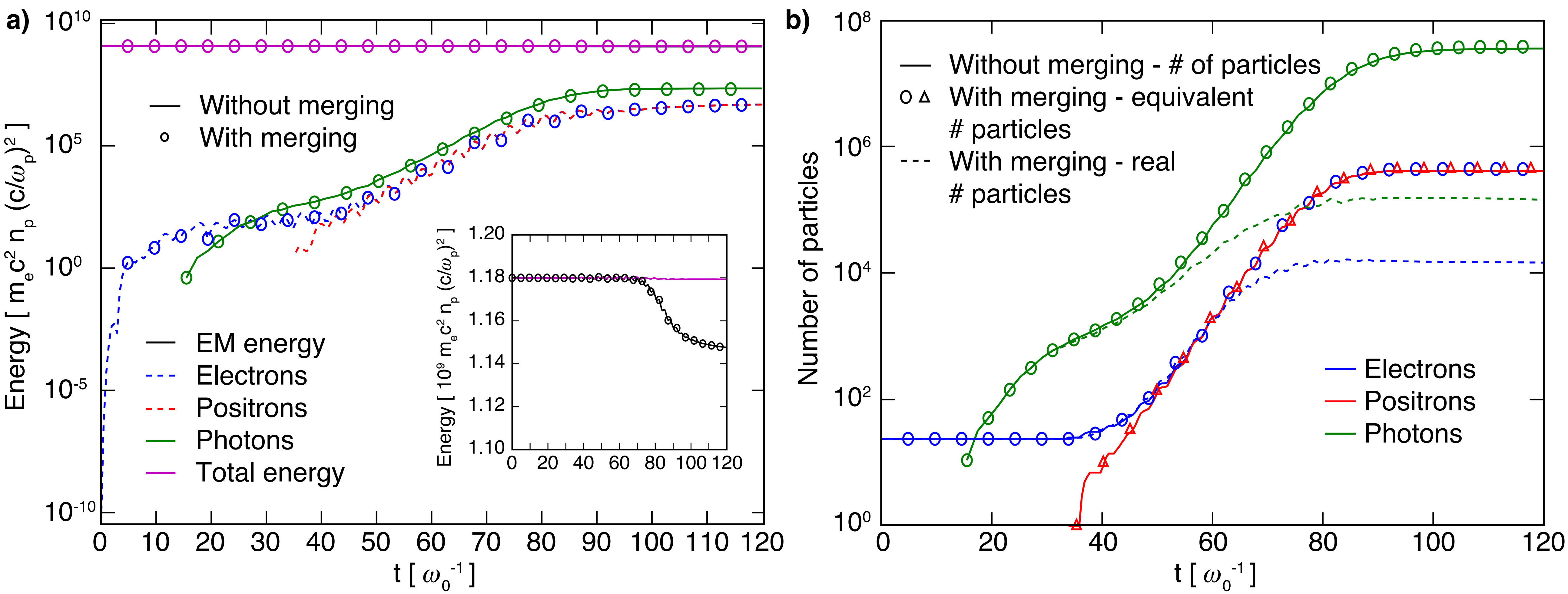


Equations to satisfy

$$w_t = w_a + w_b \ ,$$
$$\vec{p}_t = w_a \vec{p}_a + w_b \vec{p}_b$$
$$\epsilon_t = w_a \epsilon_a + w_b \epsilon_b$$

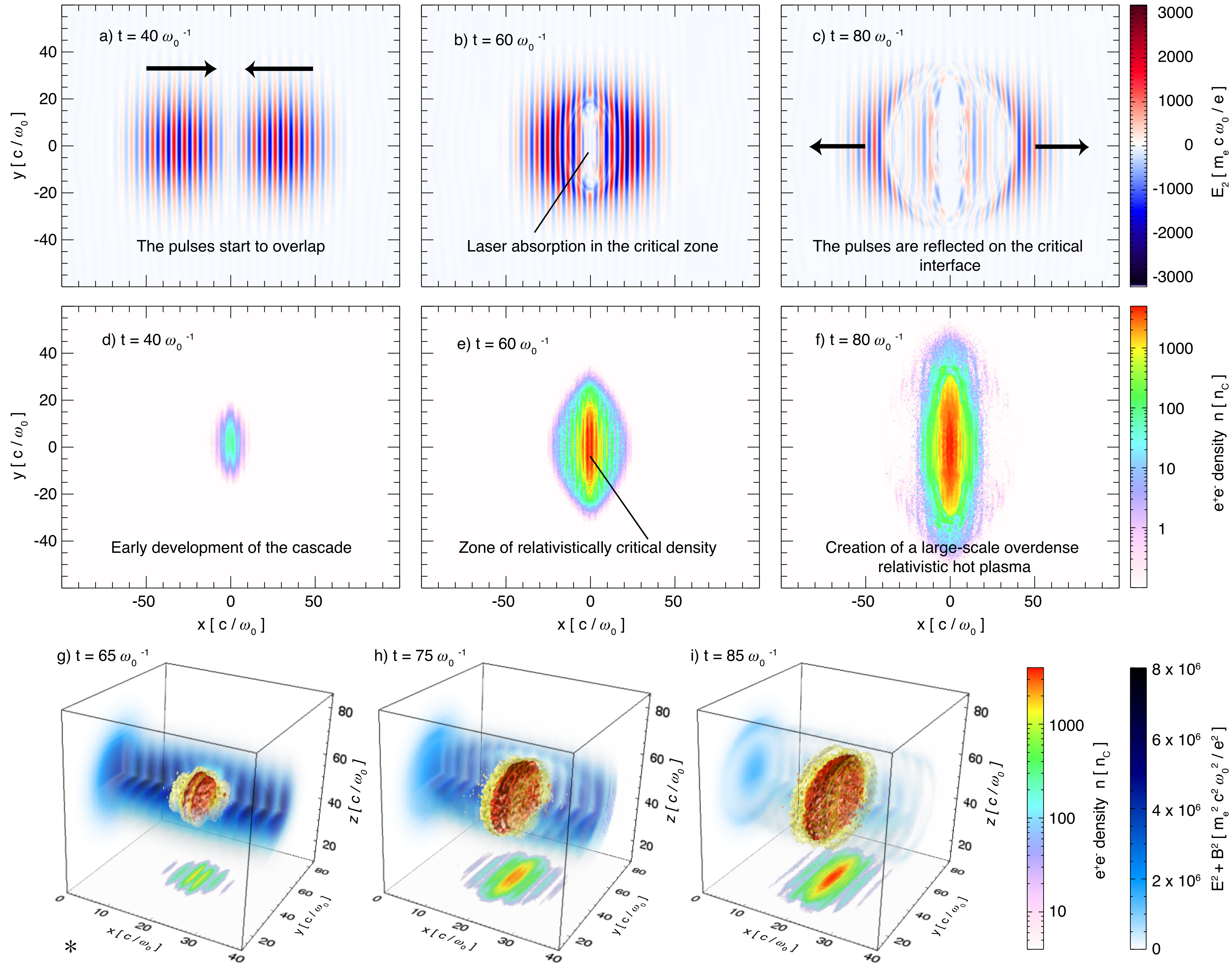
* M. Vranic, et al., CPC. 191, 65-73 (**2015**)

Growth of the number of particles



* M.Vranic, et al., CPC. 191, 65-73 (**2015**)

Interaction between self-consistent created pair plasma and the lasers



*

T. Grismayer et al., *PRE* 95, 023210 (**2017**)

T. Grismayer et al., *PoP* 23, 056706 (**2016**)

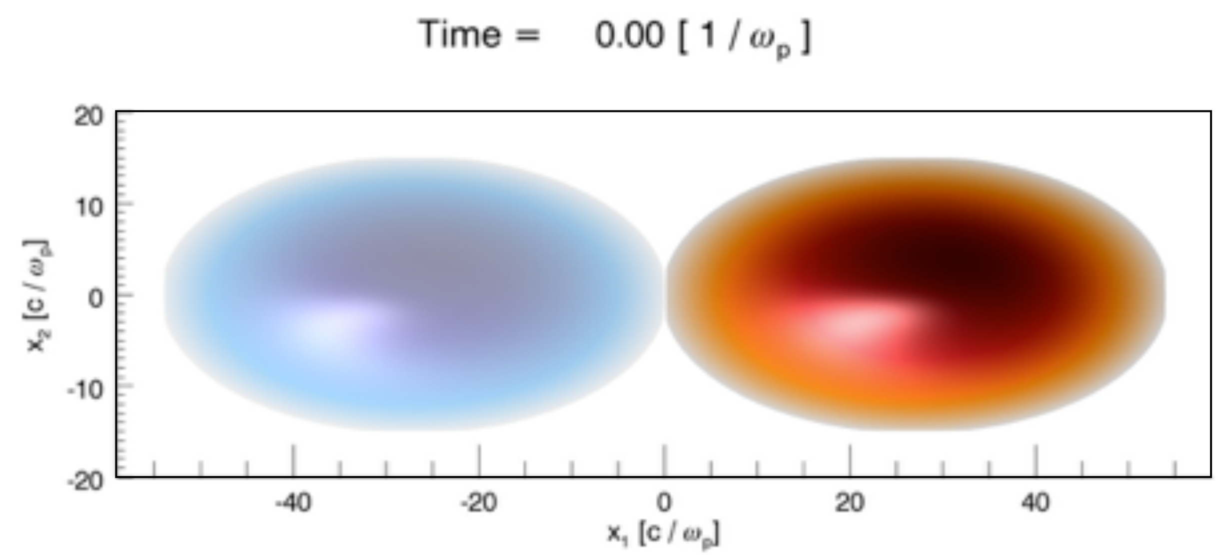
Disruption regimes for e^-e^+ colliding beams

Disruption parameter

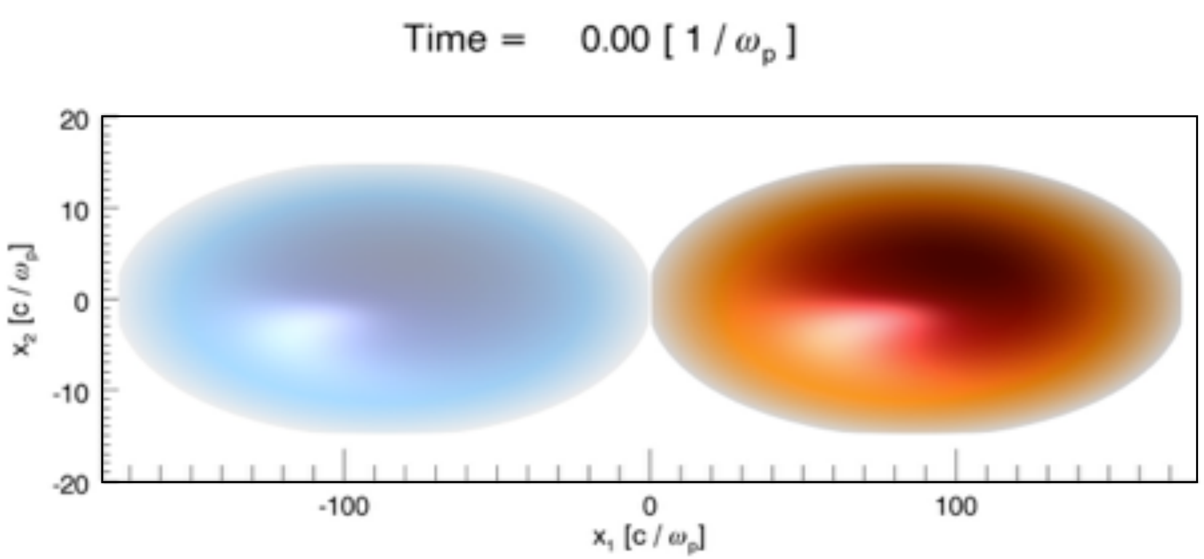
The disruption parameter relates to the number of pinching of the beams during their interaction time and identifies three different regimes

$$D = \frac{r_e N \sigma_z}{\gamma \sigma_0^2}$$

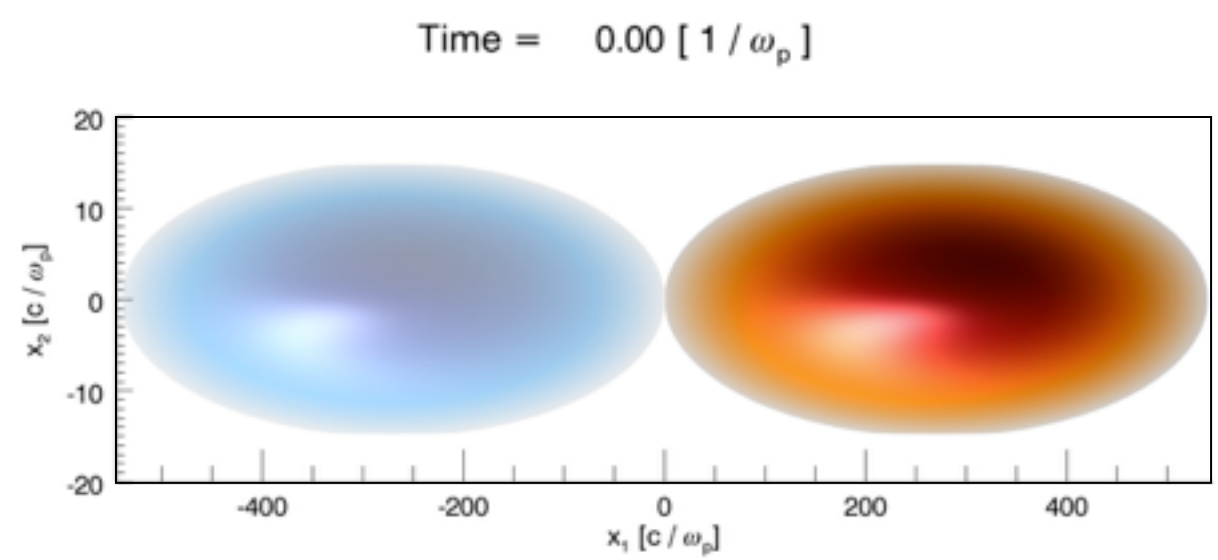
Low disruption regime D<1



Transition regime 1<D<10



Confinement regime D>10

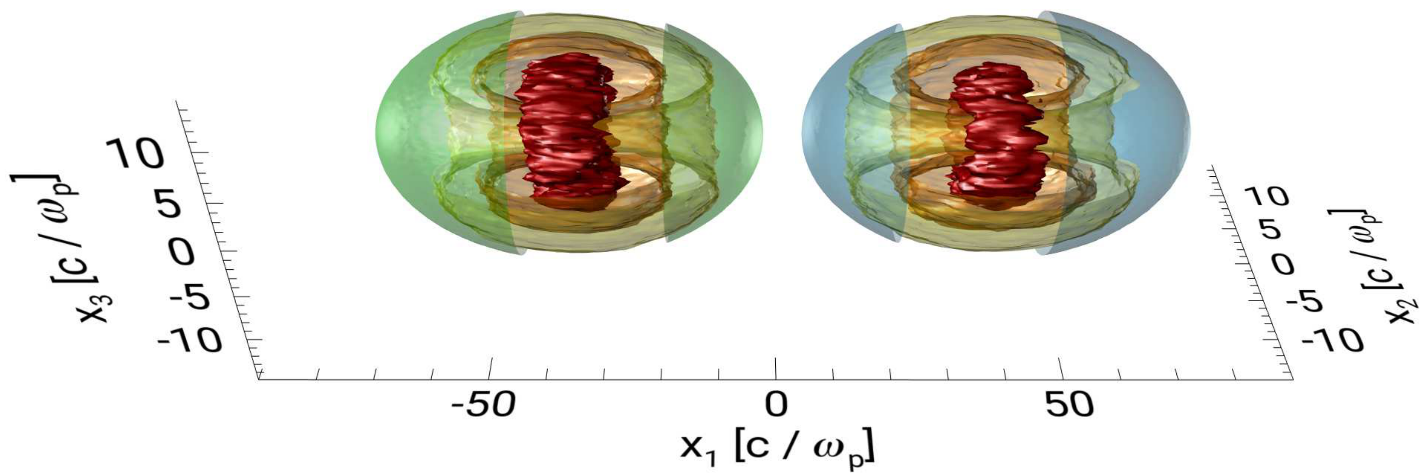


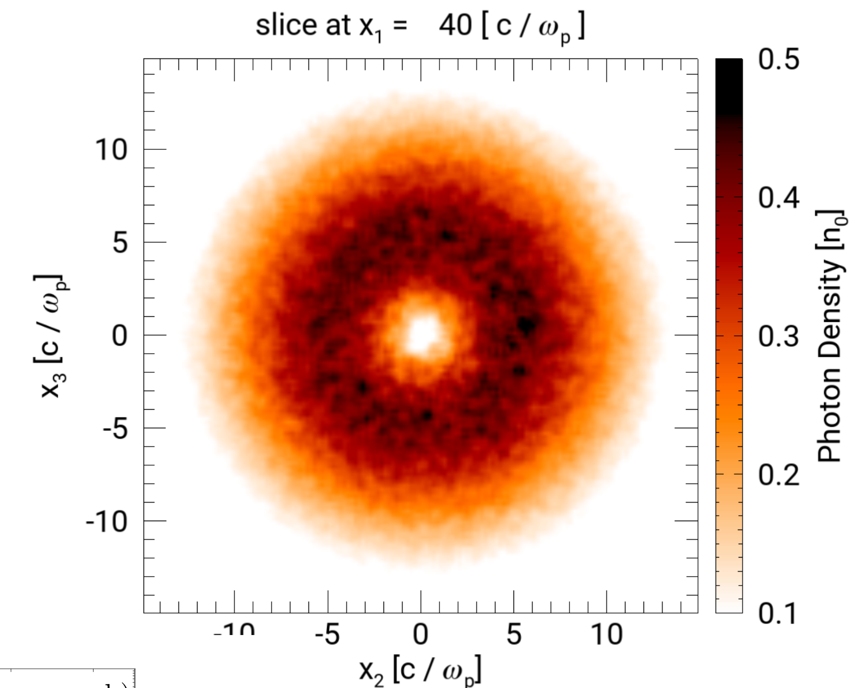
Electron beam density

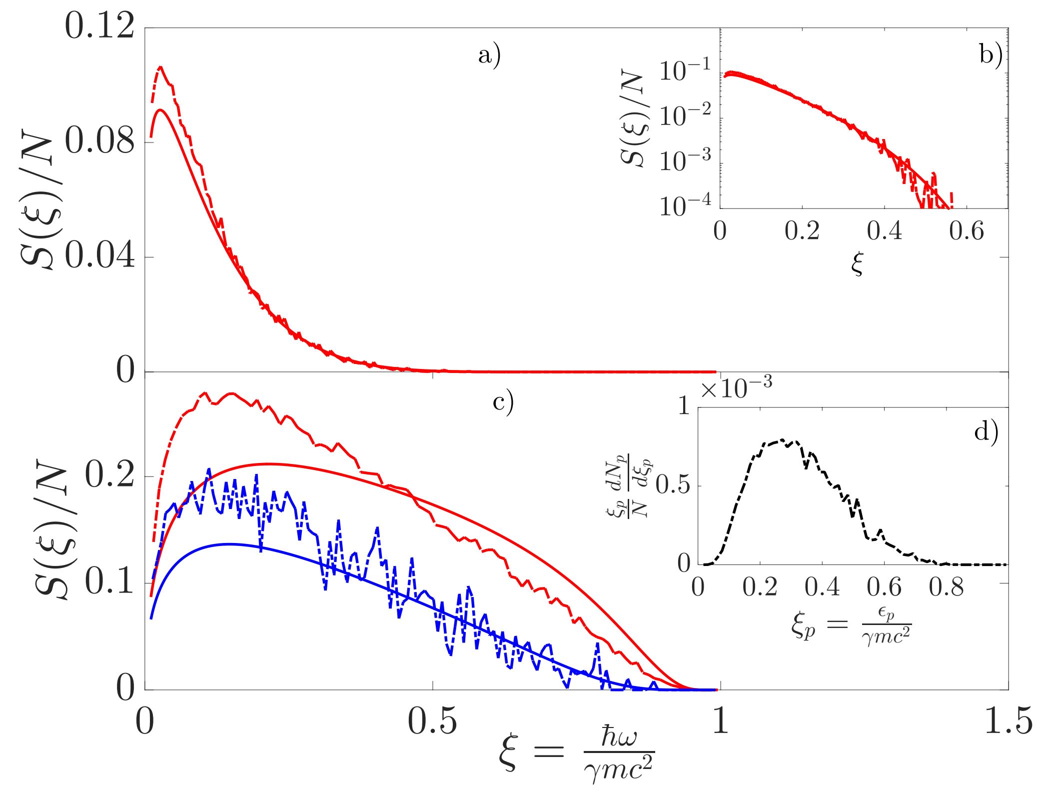
Positron beam density

Disruption regimes for e^-e^+ colliding beams

Photon spectrum



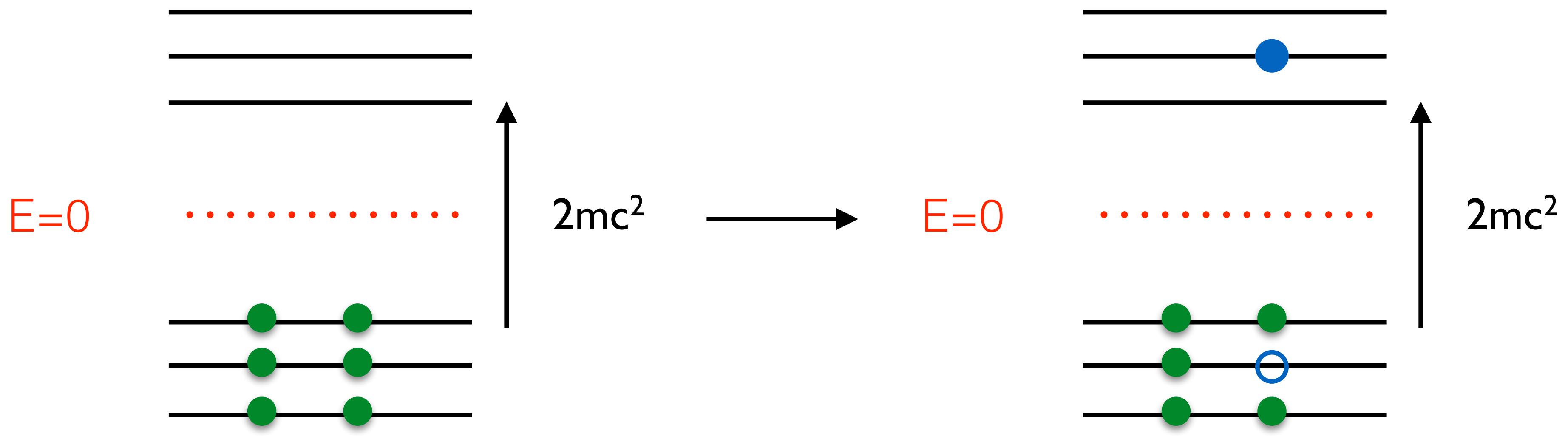




Dirac's infinite sea of electrons with negative energy

P. Dirac

"A hole, if there were one, would be a new kind of particle, unknown to experimental physics, having the same mass and opposite charge to an electron"



- From Dirac's hole theory, it became clear to Heisenberg that the vacuum would become nonlinear due to quantum fluctuations

→

- 1936: Heisenberg-Euler (HE) QED corrections to Maxwell's equations*

- H.Euler's PhD thesis topic.

The behaviour of EM fields, such as light, may be affected by quantum fluctuations of the vacuum

Heisenberg-Euler QED corrections

Heisenberg-Euler corrections to Maxwell's Equations

- Electron-positron fluctuations give rise to an effective polarization and magnetization of the vacuum which can be treated in an effective form as corrections to Maxwell's equations*.

$$\frac{\partial \vec{B}}{\partial t} + \vec{\nabla} \times \vec{E} = 0 \qquad \vec{\nabla} \cdot B = 0$$

$$\vec{\nabla} \times \vec{H} - \frac{\partial \vec{D}}{\partial t} = 0 \qquad \vec{\nabla} \cdot D = 0$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{B} = \mu_0 \vec{H} + \vec{M}$$

- With the effective vacuum polarization and magnetization

$$\vec{P} = 2\xi \left[2(E^2 - c^2B^2)\vec{E} + 7(\vec{E} \cdot \vec{B})\vec{B} \right]$$

$$\vec{M} = -2\xi \left[(2(E^2 - c^2B^2)\vec{B} - 7(\vec{E} \cdot \vec{B})\vec{E} \right]$$

- **Relevance for extreme astrophysical scenarios?**

J.Pétri, Mon. Not. Roy. Astron. Soc (**2015**)

- **Unprecedented intensities will allow to probe the quantum vacuum!**
  What laser properties will be affected?

A. Di Piazza et.al, Rev. Mod. Phys. 84, 1177–1228 (**2012**).

- **Extract observable consequences o fundamental QED predictions.**

* W. Heisenberg and H. Euler, Z. Physik 98, 714 (**1936**).

P. Carneiro et al., arxiv: 1607.04224 (**2016**) A.P. Domenech and H. Ruhl arxiv:1607.00253 (**2016**)

Nonlinear Yee Solver: numerical algorithm

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Δt

Linear Yee Scheme*
(predicted quantities)

Interpolate fields in space
$E_{(i,j)+1/2}, B_{i,j}$

EM Invariants

Polarisation & Magnetisation

Recursive loop for Convergence

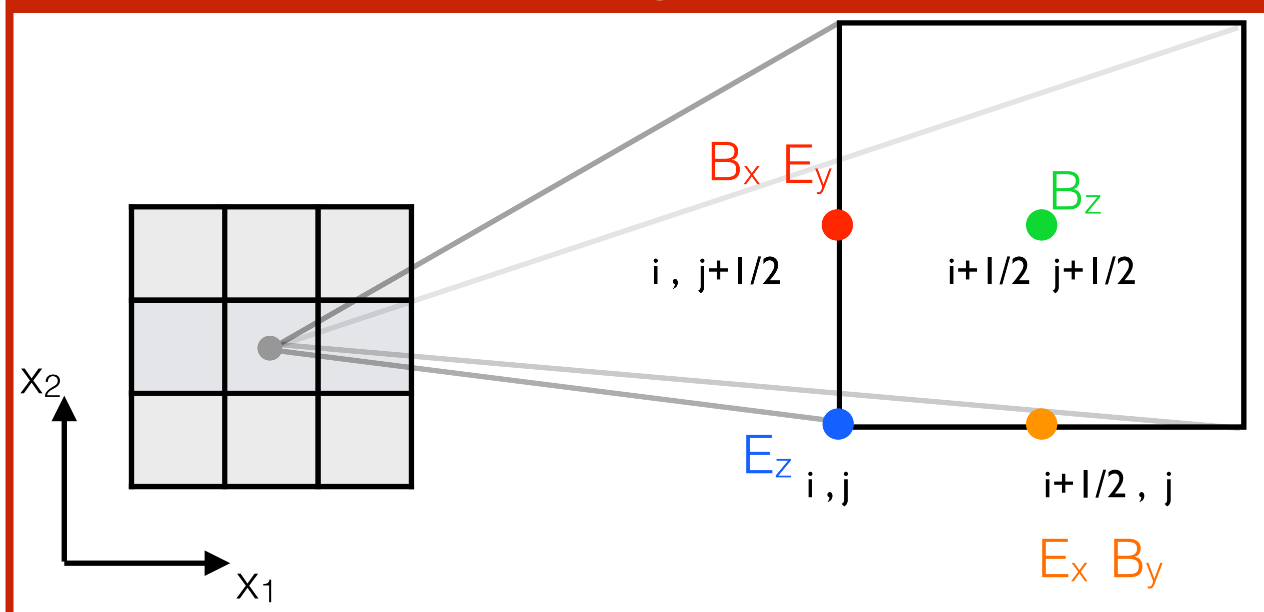
Advance E field w/ nonlinearities
(modified Ampere)

Corrected Faraday law

Nonlinear Solver**

- Ampère's law is corrected by nonlinear polarization & magnetization.
- EM invariants couple all components of all fields → necessary to calculate them at all grid points.
- Temporally the loss of linearity does not allow fields to be straightforwardly advanced → Predictor method necessary

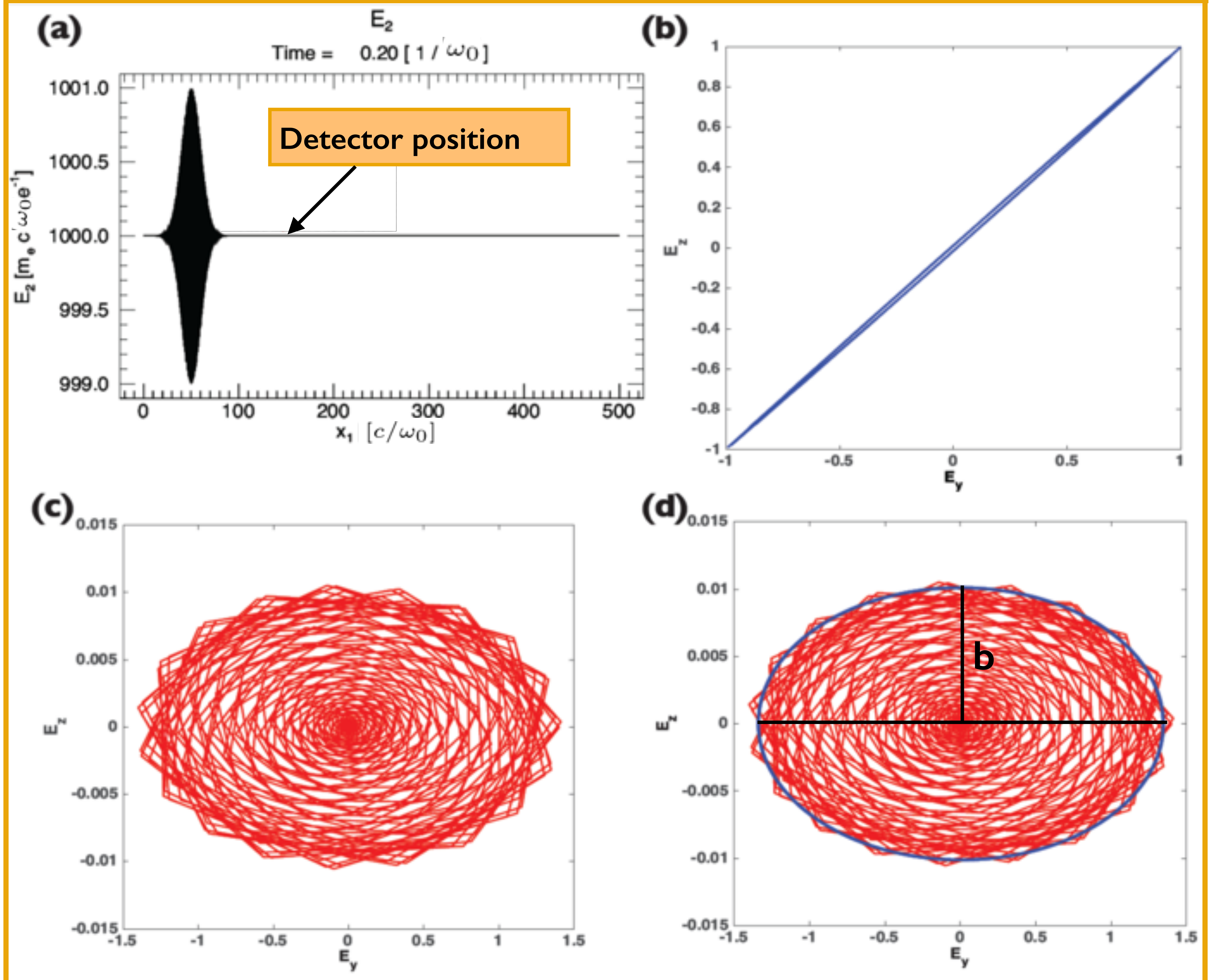
2D Yee grid cell



*K. S. Yee, IEEE Trans. Antennas Propagat., 14, 302–307, **1966**. **P. Carneiro et al., arxiv: 1607.04224 (**2016**)

Vacuum Birefringence

Detector used for data analysis (e.x: Static Field)



Features

- Subtract static components
- Rotate the resulting ellipse by a definite angle (useful for pulses with different polarization angles)

For transverse profiles:

- Able to run the diagnostic in the y direction

Outputs:

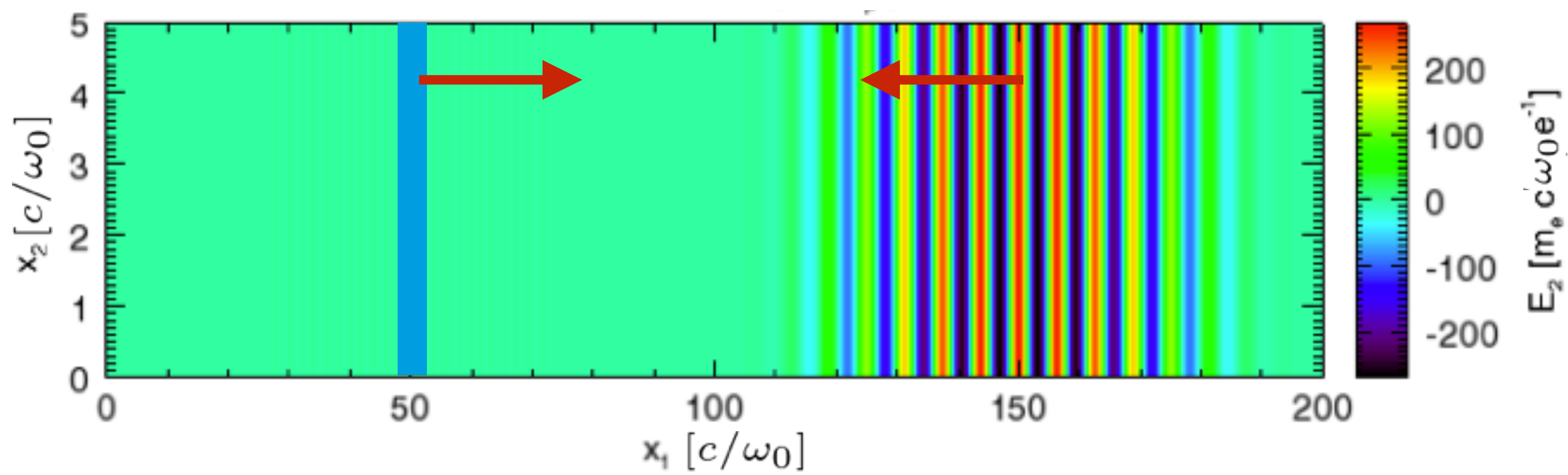
- Value of b (to evaluate the ellipticity)
- Transverse pump profile

E. Brezin and C. Itzykson, *PRD* 3, 619 (**1971**)

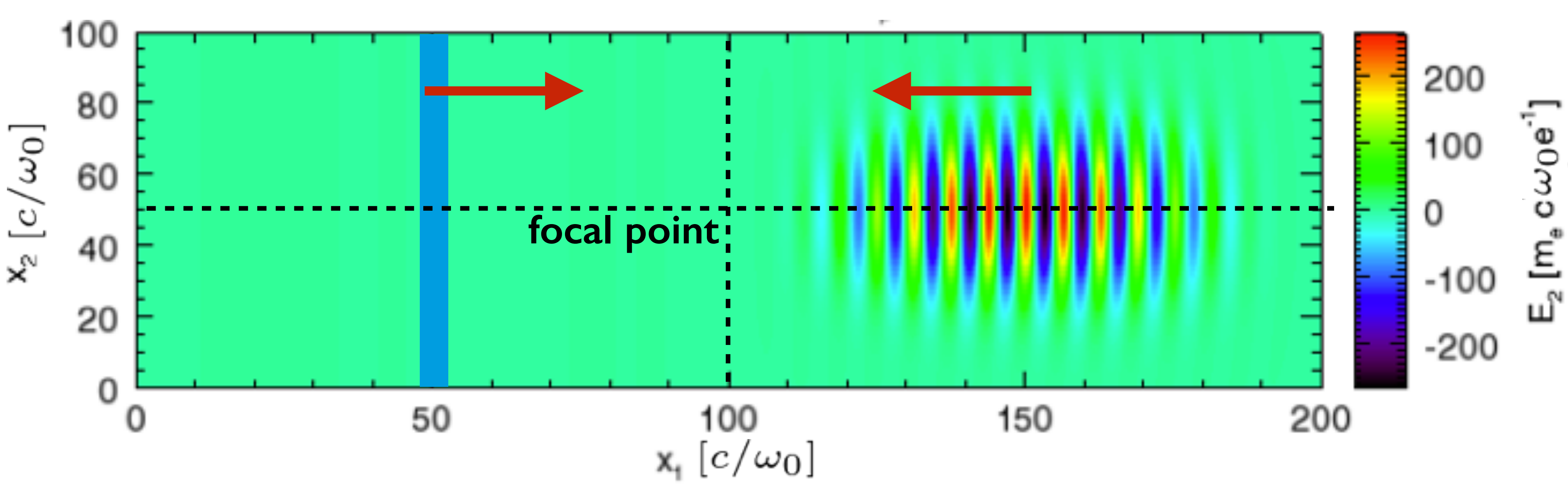
Vacuum Birefringence

Plane Wave transverse pump profile vs Gaussian pump beam profile

Plane Wave Transverse Pump Profile



Gaussian Pump Beam Profile



Pump pulse

$$I_0 = 10^{23}\ [W/cm^2]$$

$$\lambda_0 = 1\ [\mu m]$$

Gaussian transverse profile

$$W_0 = 20\ [c/\omega_0]$$

Probe pulse

$$I_p = 10^{18}\ [W/cm^2]$$

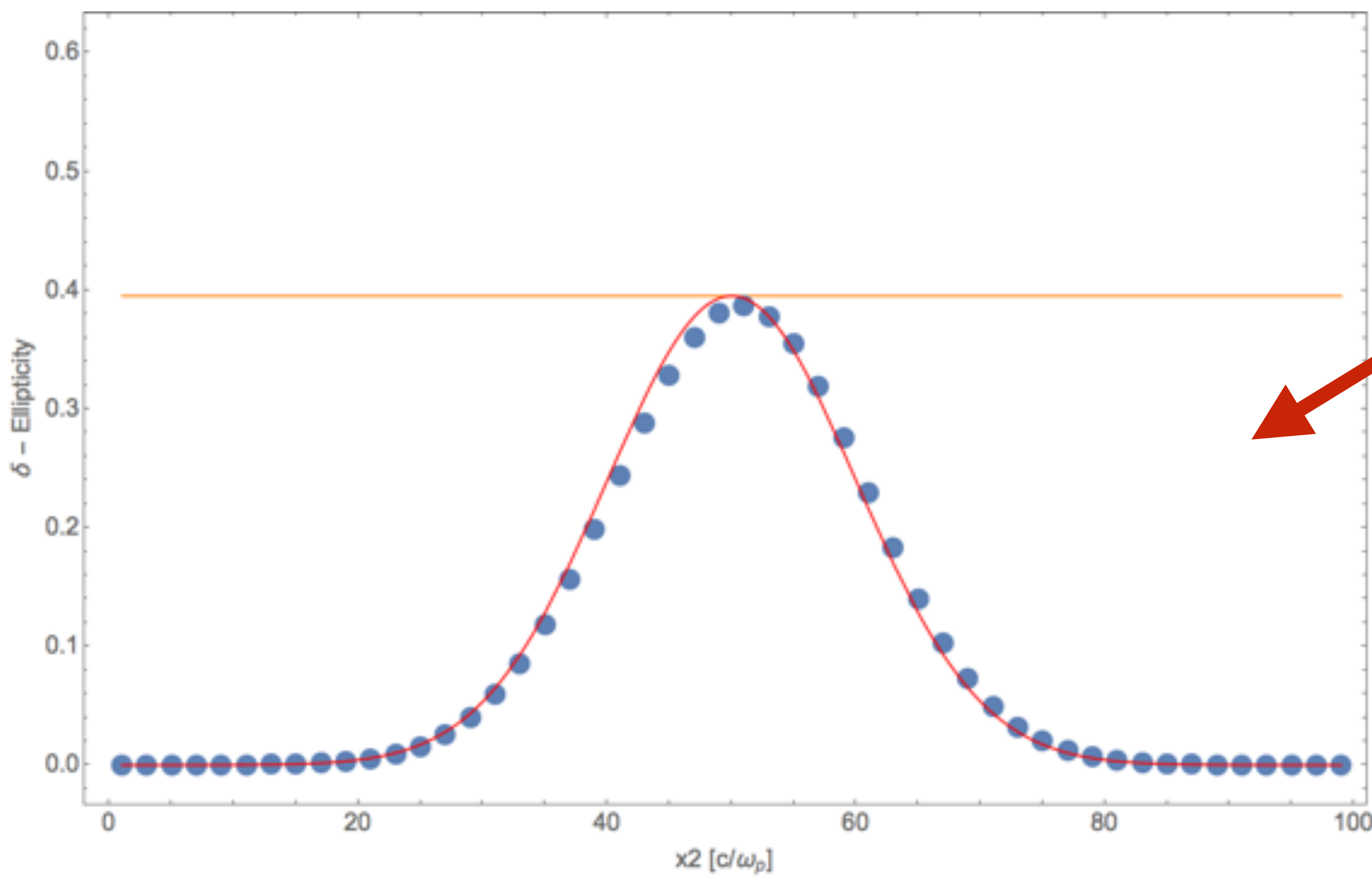
$$\lambda_p = 10\ [nm]$$

$$pol_p = \frac{\pi}{4}[rad]$$

V. Dinu et al., *PRD* 89, 125003 (**2014**)
S. Bragin et al., arxiv: 1704.05234 (**2017**)   F. Karbstein et al., *PRD* 92, 071301 (**2015**)

Vacuum Birefringence

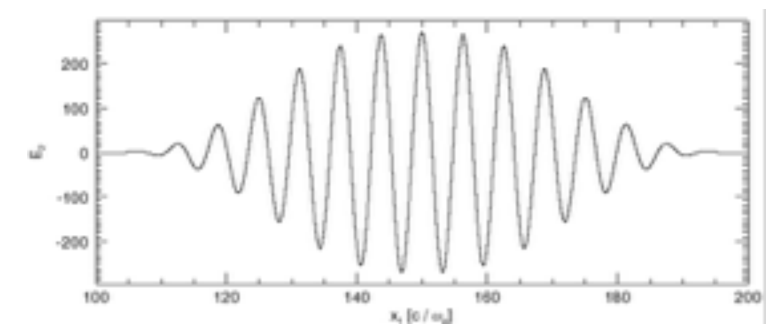
Plane Wave transverse pump profile vs Gaussian pump beam profile



- Numerical result - Gaussian profile
- Theoretical result - Gaussian profile*
- Theoretical result - Plane wave profile

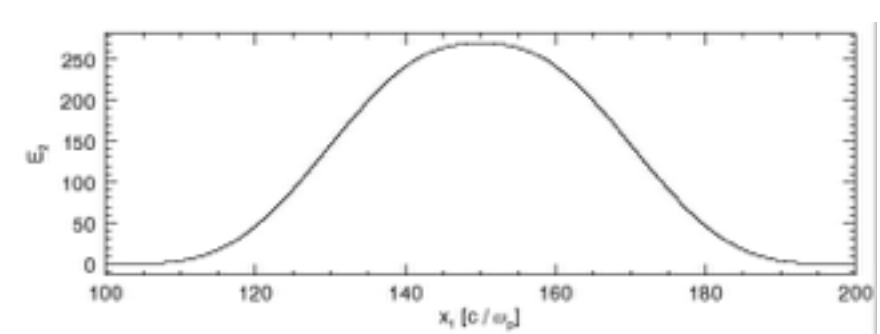
Oscillating gaussian profile

$$\delta = \frac{3}{2}\sqrt{\pi}k_p\sigma\xi E_0^2$$



Non-oscillating gaussian profile

$$\delta = 3\sqrt{\pi}k_p\sigma\xi E_0^2$$



* V. Dinu et al., Physical Review D 90, 045025 (**2014**)

Interaction of two gaussian beams

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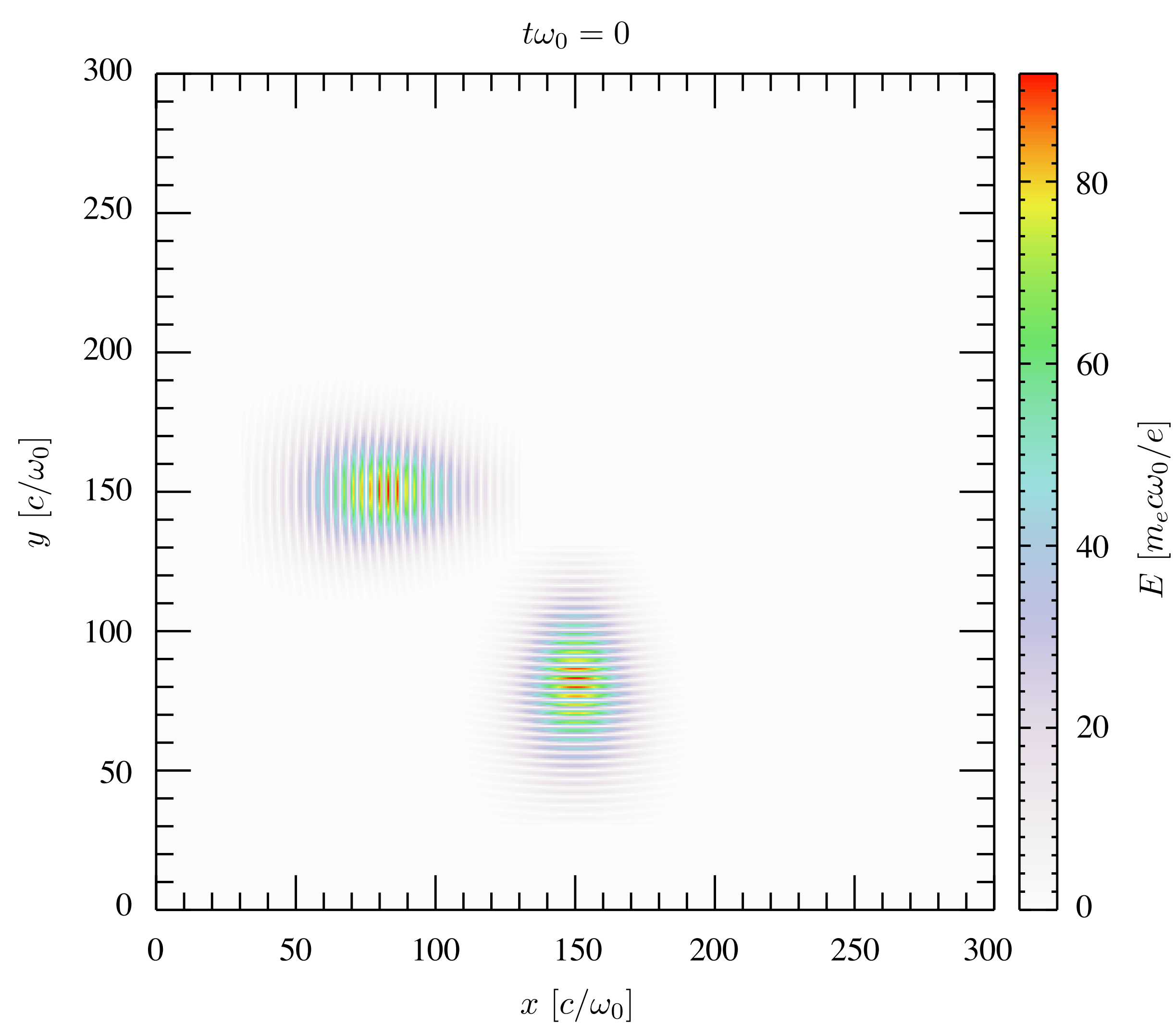
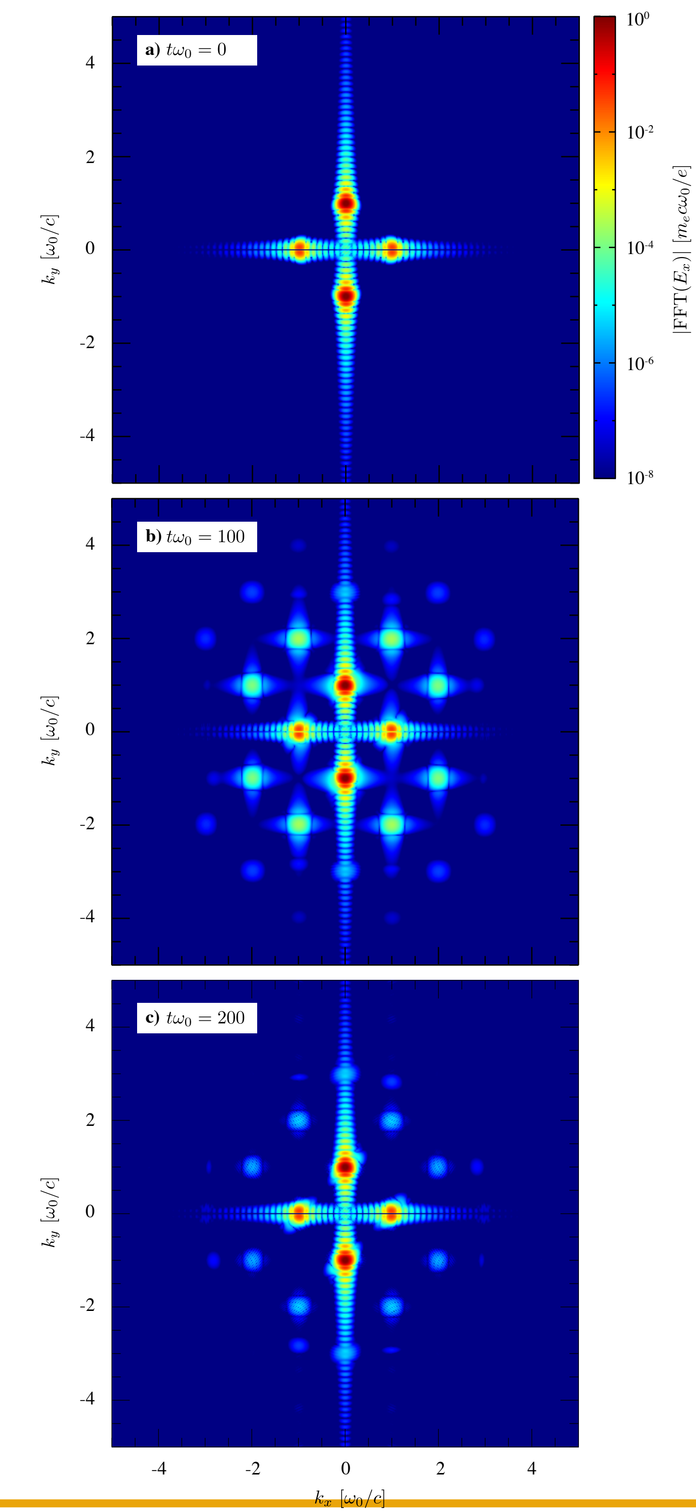
Perpendicular interaction of two gaussian beams





A. Di Piazza et al., *PRD* 72, 085005 (**2005**)

Conclusions

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New Tools to tackle a variety of extreme plasma physics problems

- Post processing Radiation & Classical Radiation Reaction (LL)
- QED module (Breit-Wheeler: photons + pairs)
- Vacuum polarization solver

The merging algorithm is critical to perform full 2d/3d QED-simulations

- We observe the interaction of a self-generated pair plasma with the lasers

These tools can be used to study extreme astrophysical scenario

- Study pulsar's pair cascades
- Other quantum processes can be included to account for curvature radiation
- Possibility to carry out self-consistent population of pulsar magnetosphere