Multi-dimensional shearing modules in OSIRIS 4.0

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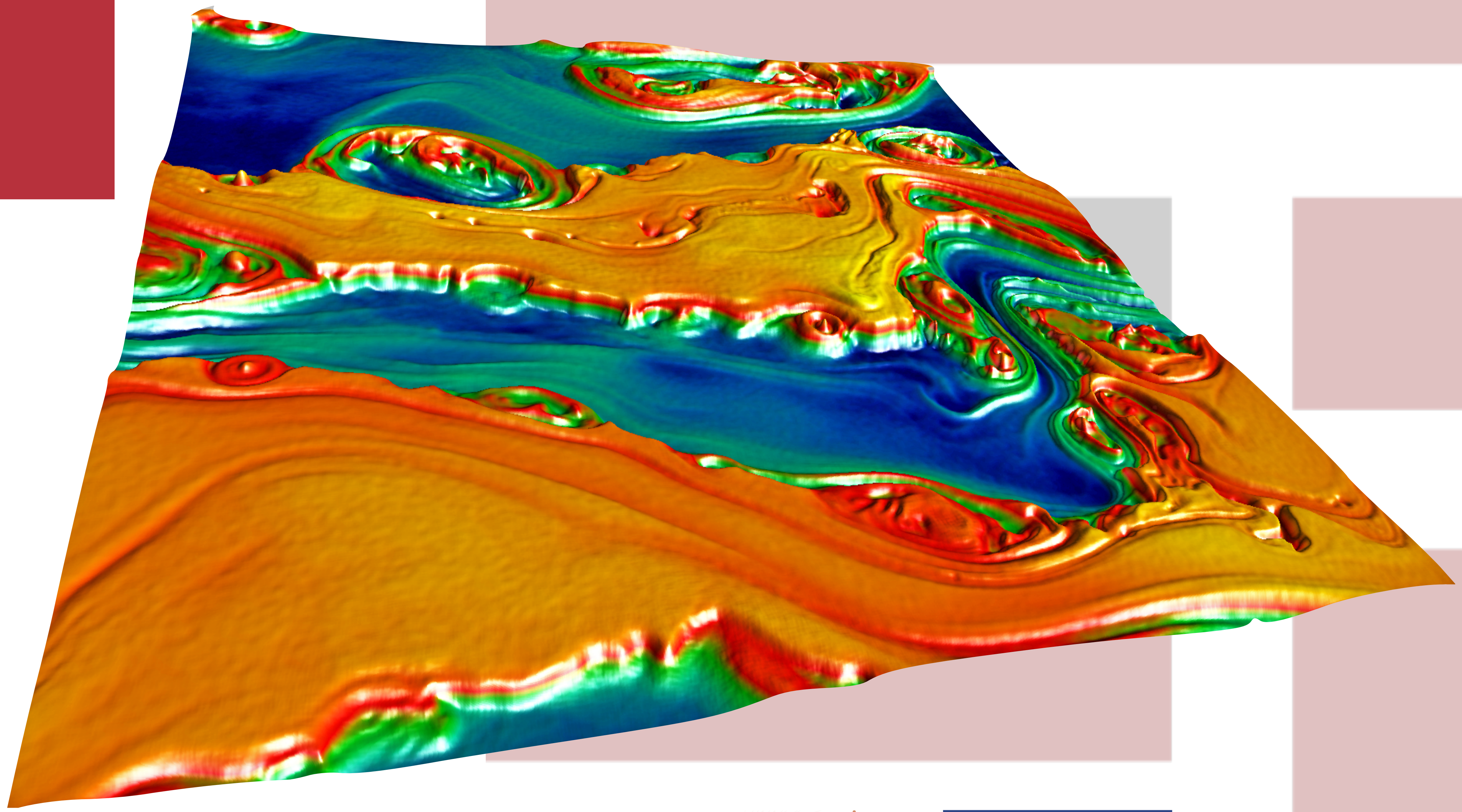

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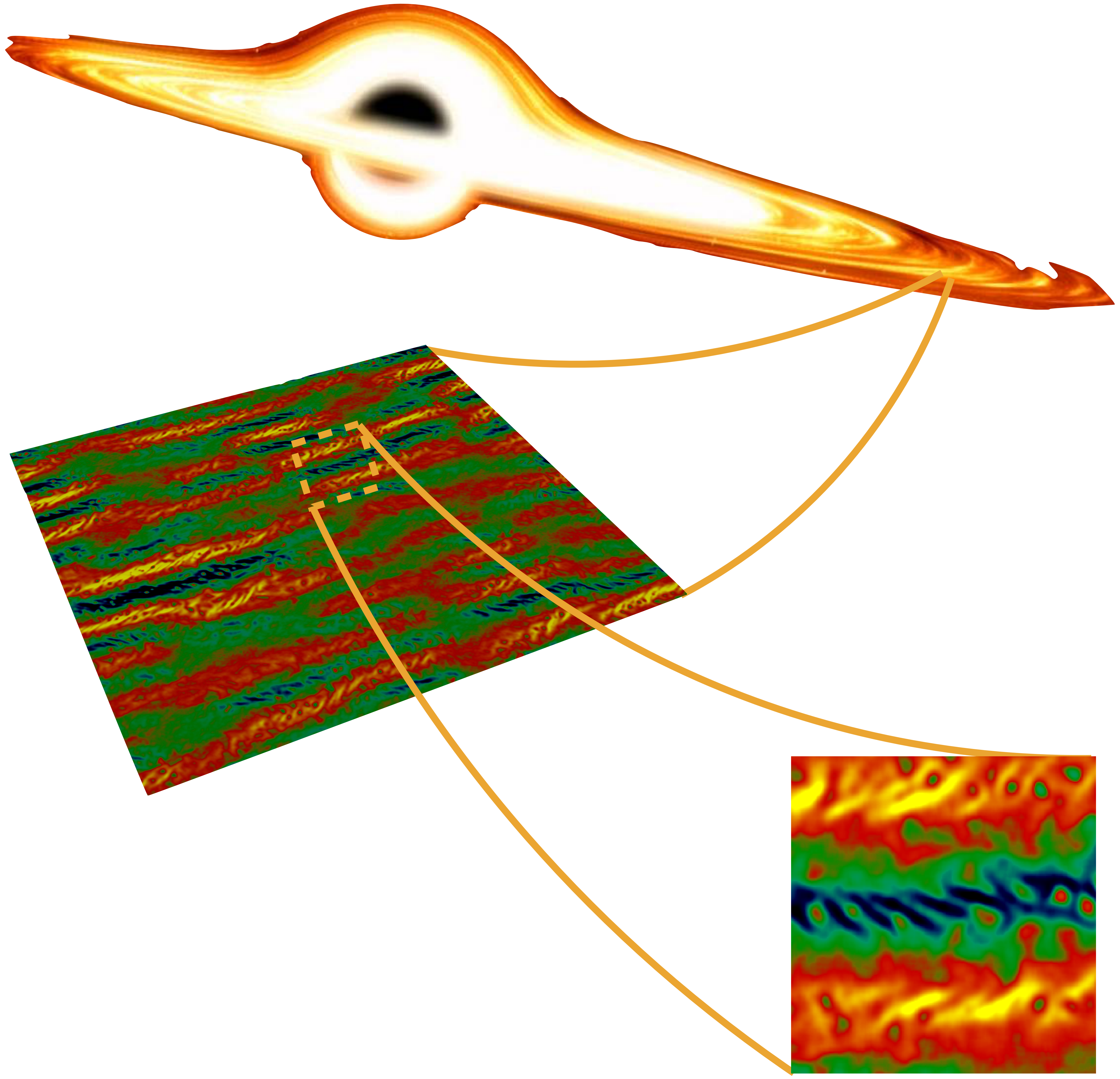
Why a shearing module in a PIC code?

Large scale astrophysical rotating system → Accretion disks

- Local kinetic behaviour

Generation of pressure anisotropy in collisionless plasmas

- Mirror instability
- Firehose instability
- Whistler instability
- …



*Simulation results obtained using OSIRIS 4.0

Outline

Necessity of shear module
- Collisionless magneto rotational instability (MRI)
- Study of pressure anisotropy

2D shearing co-rotating framework
- Modified Maxwell's and motion equations
- Physical benchmark: 1D linear MRI

3D co-rotating framework + shearing periodic b.c.
- The 3D shearing co-rotating framework does not work
- Description of the shearing periodic b.c.
- Benchmark: E_1 behaviour at the boundary

Conclusion & Future works

2D shearing co-rotating framework

Modified Maxwell's and motion equations

Shearing co-rotating framework equations

$$\frac{\partial \vec{B}(\vec{r},t)}{\partial t} = -\nabla \times \vec{E}(\vec{r},t) - \frac{3}{2}\alpha B_x \hat{y}$$

$$\frac{\partial \vec{E}(\vec{r},t)}{\partial t} = \nabla \times \vec{B} - 4\pi \vec{J} - \frac{3}{2}\alpha E_x \hat{y}$$

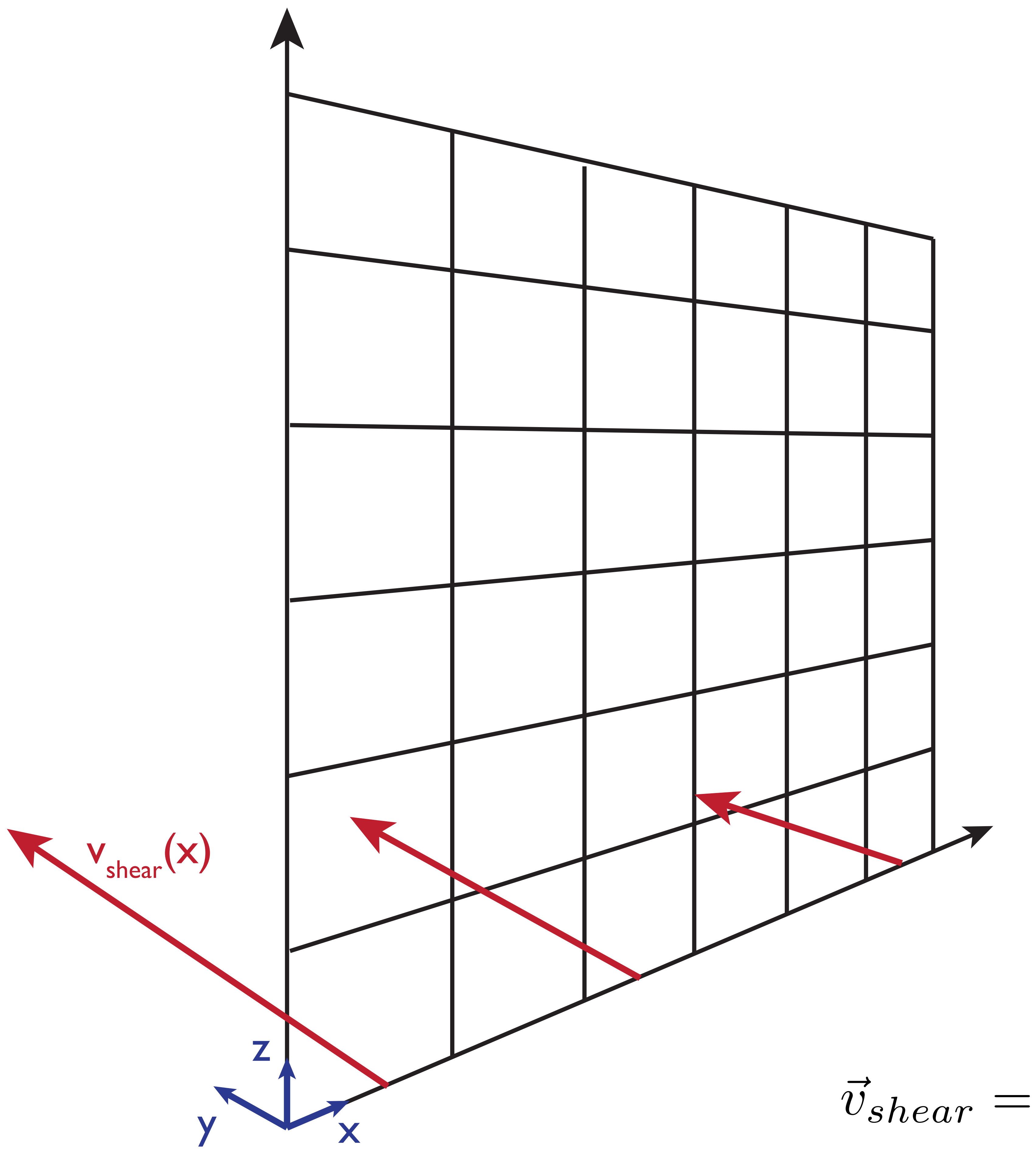
$$\frac{d\vec{p}}{dt} = 2\alpha p_y \hat{x} - \frac{1}{2}\alpha p_x \hat{y} + q\left(\vec{E} + \frac{\vec{u}}{c} \times \vec{B}\right)$$

Validity of the model in the limits:

- **non-relativistic** limit $v_0 = \alpha \times r_0 << c$
- small box approximation $L << r_0$

M.A. Riquelme, et al., ApJ 755, 50 (2012)

2D shearing simulation box



$$\vec{v}_{shear} = -\frac{3}{2}\alpha x \hat{y}$$

Scheme of implementation in OSIRIS 4.0

example of input file

EMF solver

Modified Yee solver

+

RK shear term

particle pusher

Modified Boris pusher

($B_y^* = B_y + 2\alpha * rqm * \gamma$)

+

FD shear term

shear_2D branch @ ginchingolo/osiris

dev branch @ GoLP-IST/osiris

Example of input file

```
!---global simulation parameters----
simulation
{
  algorithm = "shear",
}
          ! … (after diag_emf)
!---------shear parameter----------
shear
{
  alpha = 0.05,
}
particles
! …
```

Physical benchmark: 1D MRI dispersion relation

Comparison between numerical results and analytical model

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Analytical Expression

$$4\nu^4 + 4(1+\zeta^2)\nu^2 + \zeta^4 - 6\zeta^2 = 0$$

Simulation Parameter (quasi 1D simulation)

mass ratio

$$\frac{m_e}{m_i} = 1$$

beta parameter

$$\beta = \frac{8\pi n k_b T}{B_0^2} = 0.05$$

magnetisation

$$X = \frac{\alpha}{\omega_{ci}} = 11, 33$$

Alfvén velocity

$$\frac{v_A}{c} = \frac{B_0}{\sqrt{4\pi n m c^2}} = \frac{1}{20}$$

resolution

$$\frac{d_e}{\Delta x} = \frac{c}{\omega_{pe}\Delta x} = \frac{1}{0.007}$$

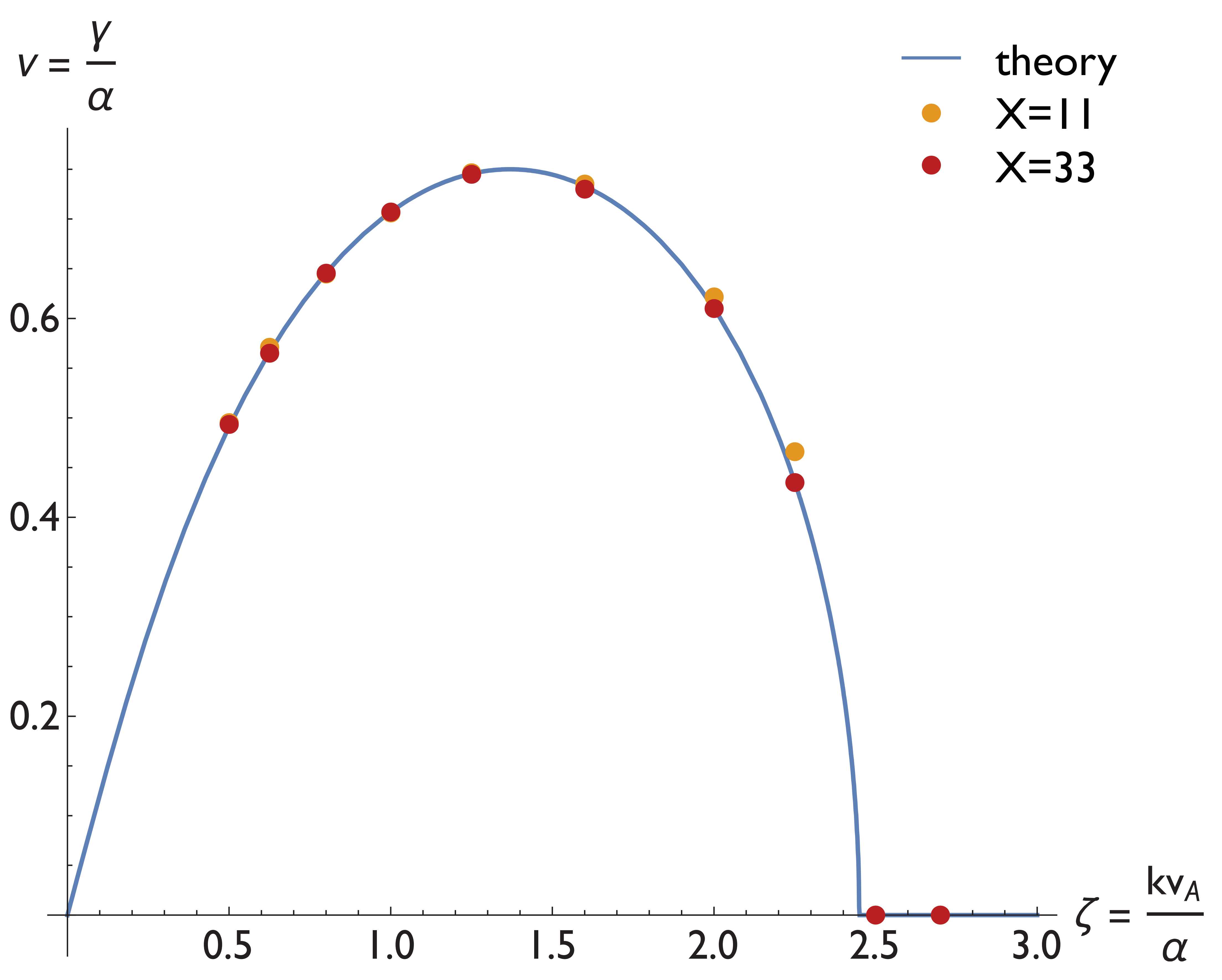
ppc

$$N_{ppc} = 1000$$

seed

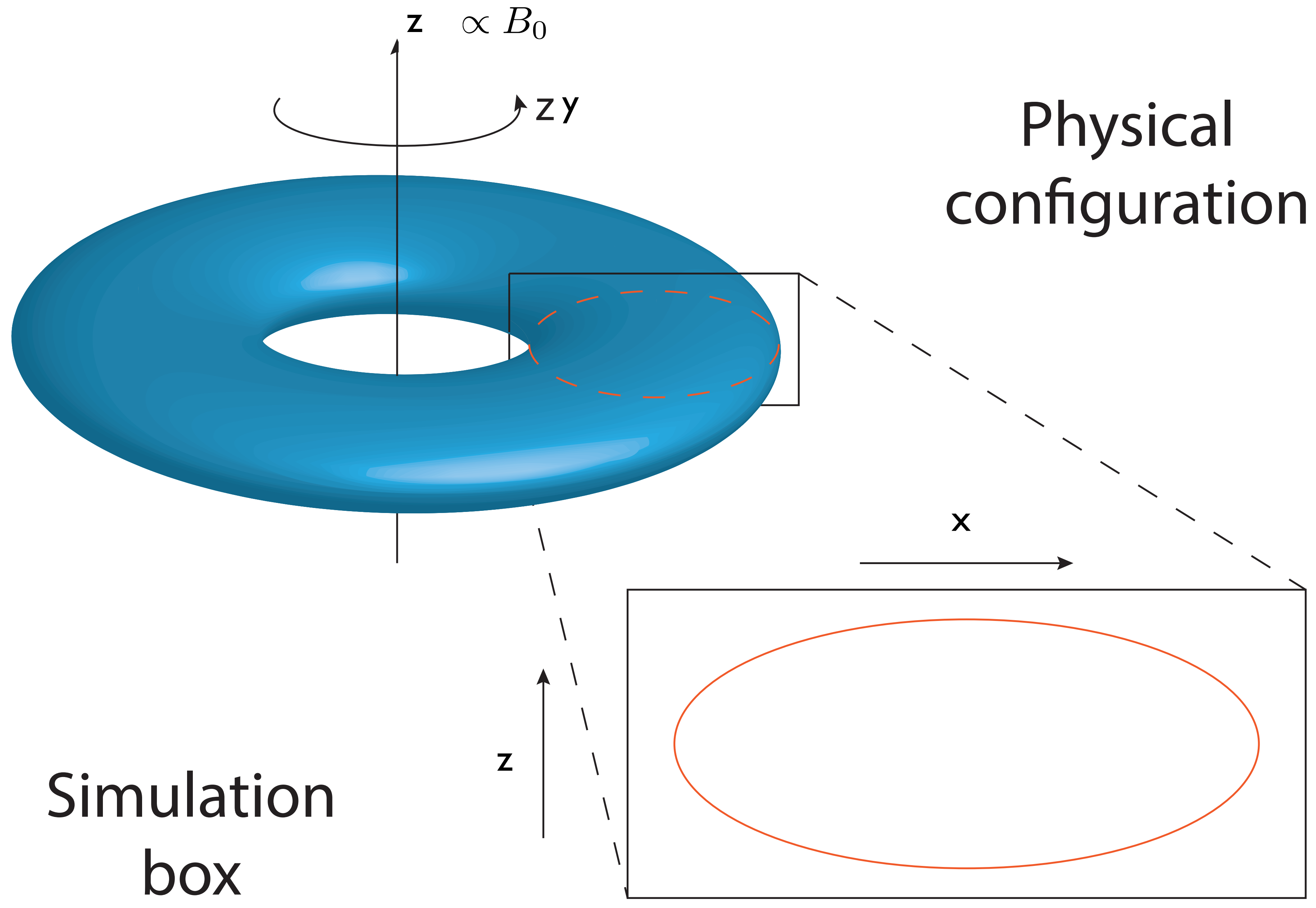
$$\frac{\vec{v}_{seed}}{c} = \frac{1}{20}\frac{v_A}{c}\sin\left(\frac{2\pi x}{L}\right)\hat{y}$$

1D MRI linear dispersion relation



2D collisionless Magnetorotational Instability

Large scale mechanism for amplification of magnetic field



$$\vec{B} = B_0 \hat{z}$$

$$\vec{v}_{shear} = -\frac{3}{2}\alpha x \hat{y}$$

Simulation Parameter

mass ratio

$$\frac{m_i}{m_e} = 1$$

beta parameter

$$\beta_0 = \frac{v_{th}^2}{v_A^2} = 100$$

magnetisation

$$X = \frac{\alpha}{\omega_{ci}} = 11$$

Alfvén velocity

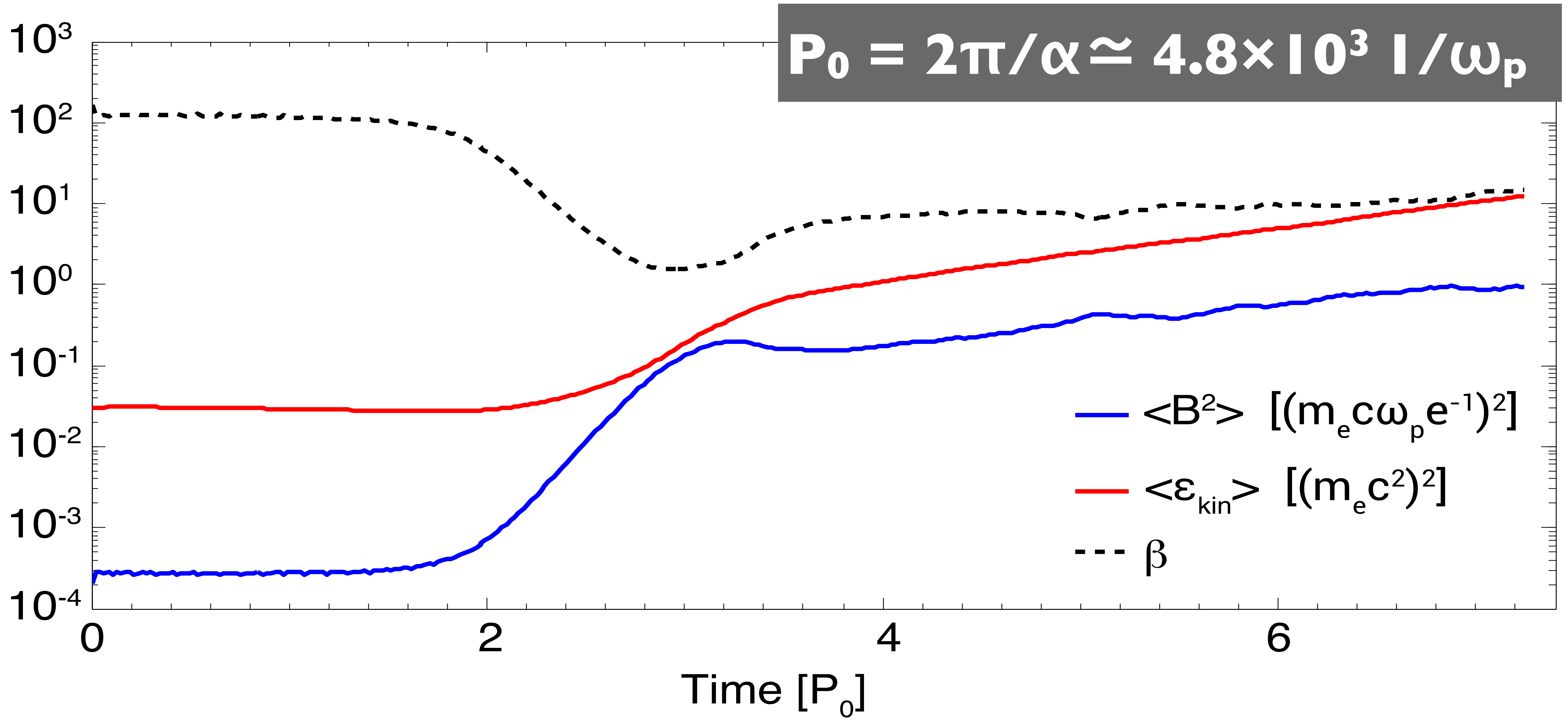
$$\frac{v_A}{c} = 0.0143$$

box size

$$16\ \lambda_0^2 \left(\lambda_0 = \frac{2\pi v_A}{\alpha}\right)$$

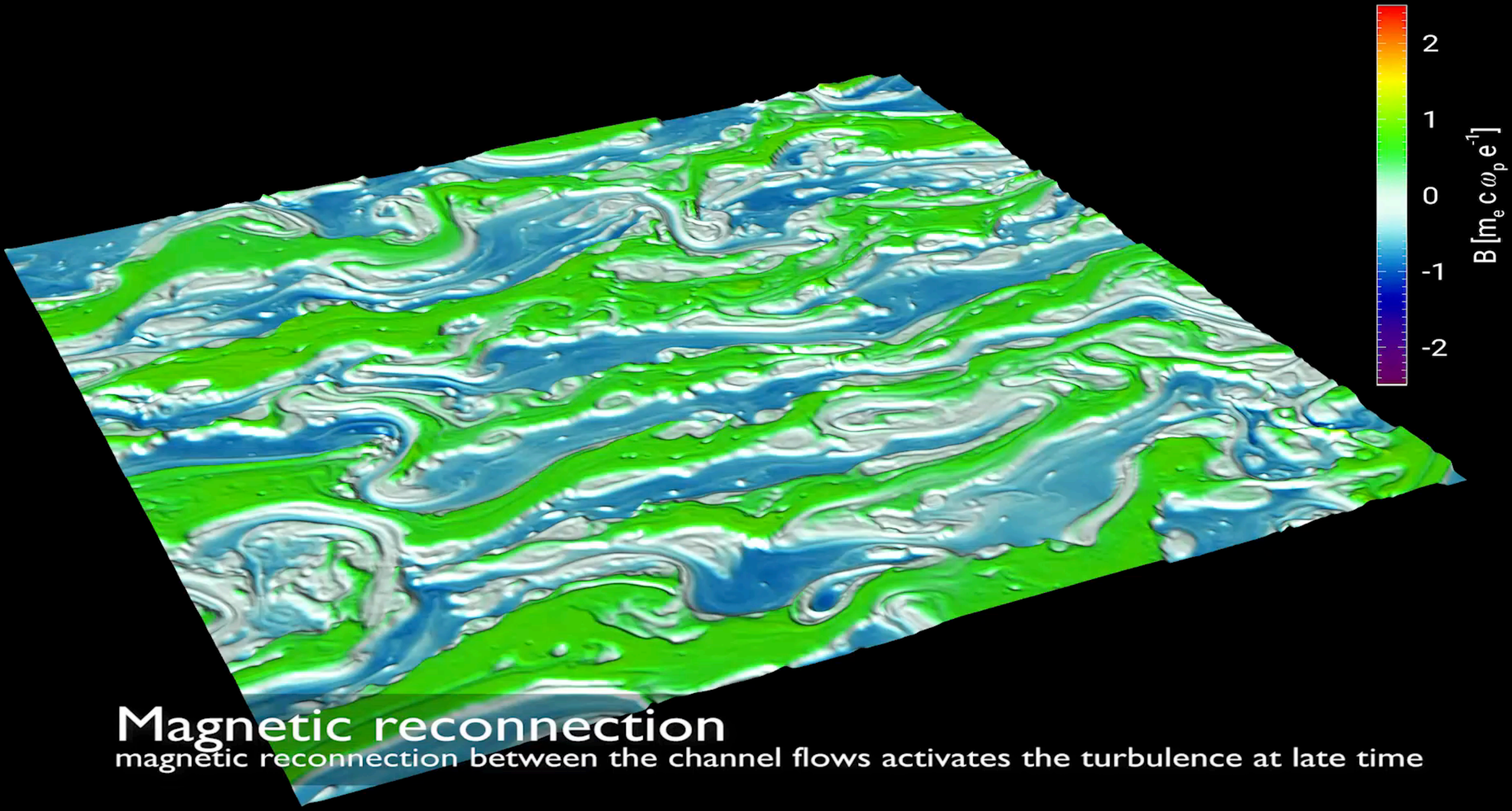
~1100x1100 c/ω_p

$P_0 = 2\pi/\alpha \simeq 4.8\times10^3\ 1/\omega_p$



Evolution of B_y in collisionless accretion disk

2D simulation of large scale collisionless Magnetorotational Instability



3D shearing co-rotating Maxwell's equations

$$\frac{\partial \vec{B}(\vec{r},t)}{\partial t} = -\nabla \times \vec{E}(\vec{r},t) - \frac{3}{2}\alpha B_x \hat{y} + \frac{3}{2}\alpha t \frac{\partial \vec{E}(\vec{r},t)}{\partial y} \times \hat{x}$$

$$\frac{\partial \vec{E}(\vec{r},t)}{\partial t} = \nabla \times \vec{B} - 4\pi \vec{J} - \frac{3}{2}\alpha E_x \hat{y} - \frac{3}{2}\alpha t \frac{\partial \vec{B}(\vec{r},t)}{\partial y} \times \hat{x}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{J} = -\frac{3}{2}\alpha t \frac{\partial J_x}{\partial y}$$

Achtung! Charge is not conserved with the standard 3D PIC algorithm!

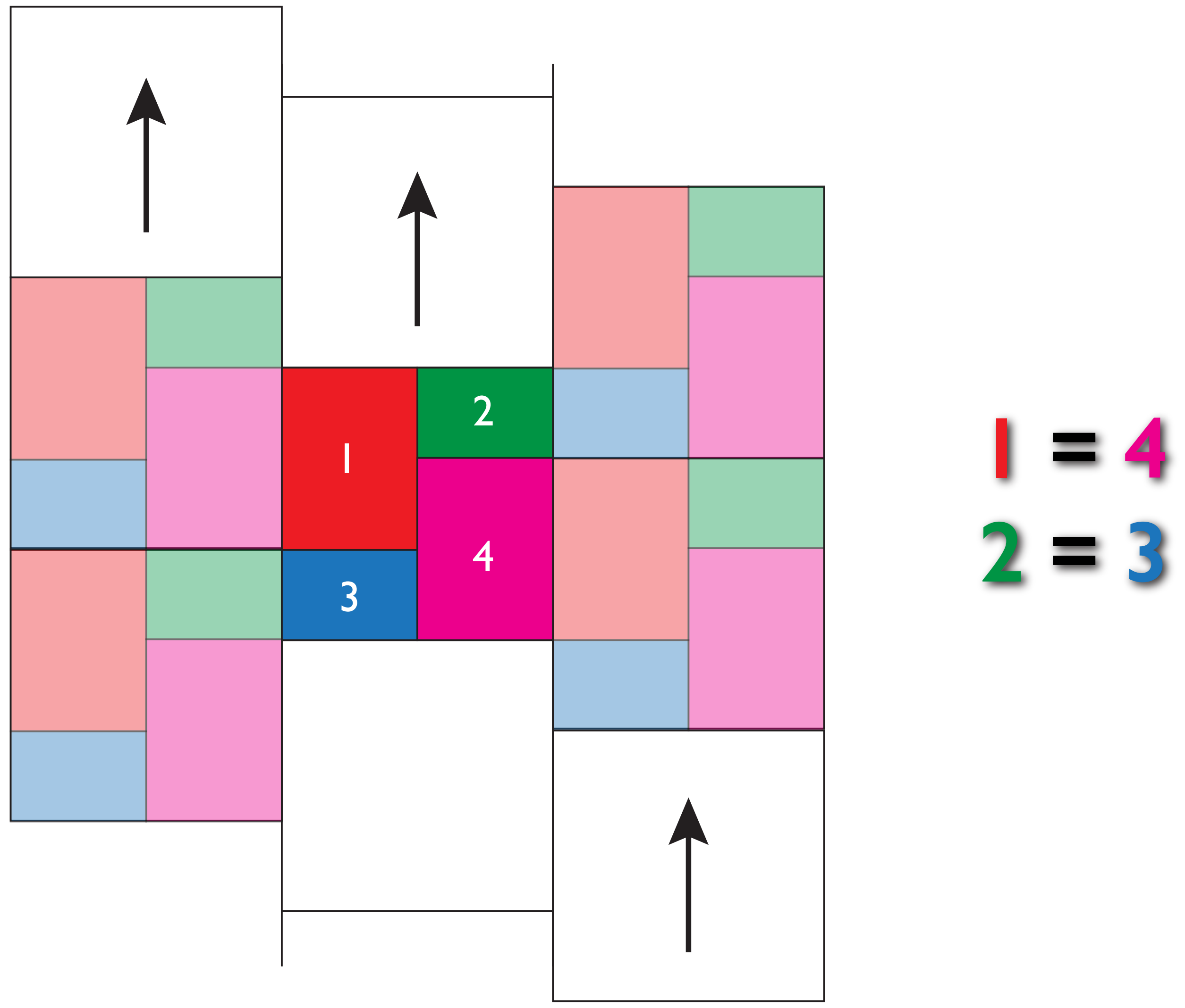
Solution: co-rotating framework

Standard Maxwell's equation* **+** $\frac{d\vec{p}}{dt} = e\left(\vec{E} + \frac{\vec{p}}{c} \times \left(\frac{\vec{B}}{m\gamma} + 2\alpha\hat{z}\right)\right)$ **+** **shear periodic b.c.**

*in the non relativistic limit

Shear periodic b.c.

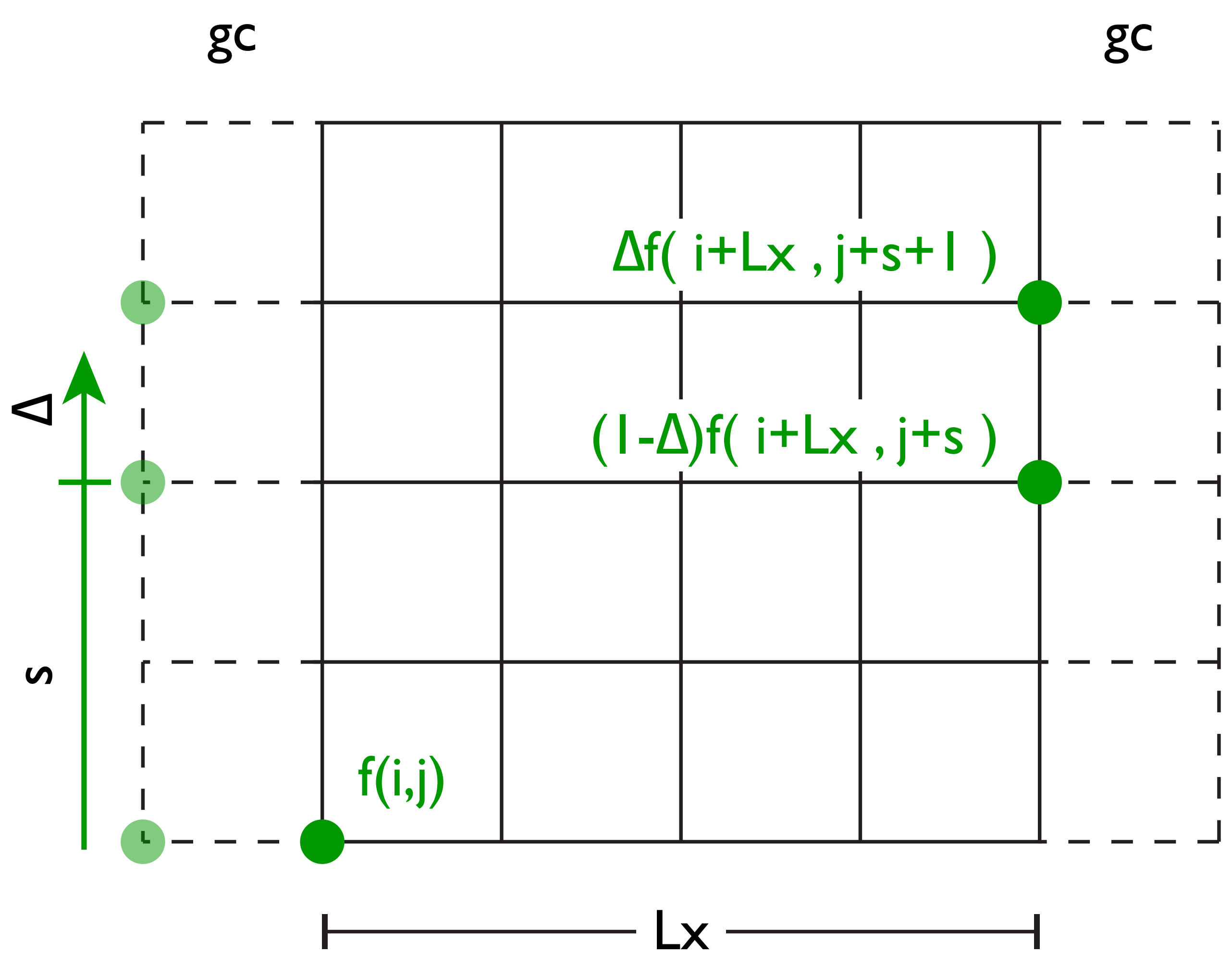
Deposition of shifted grid quantities (fields/current) at the boundary



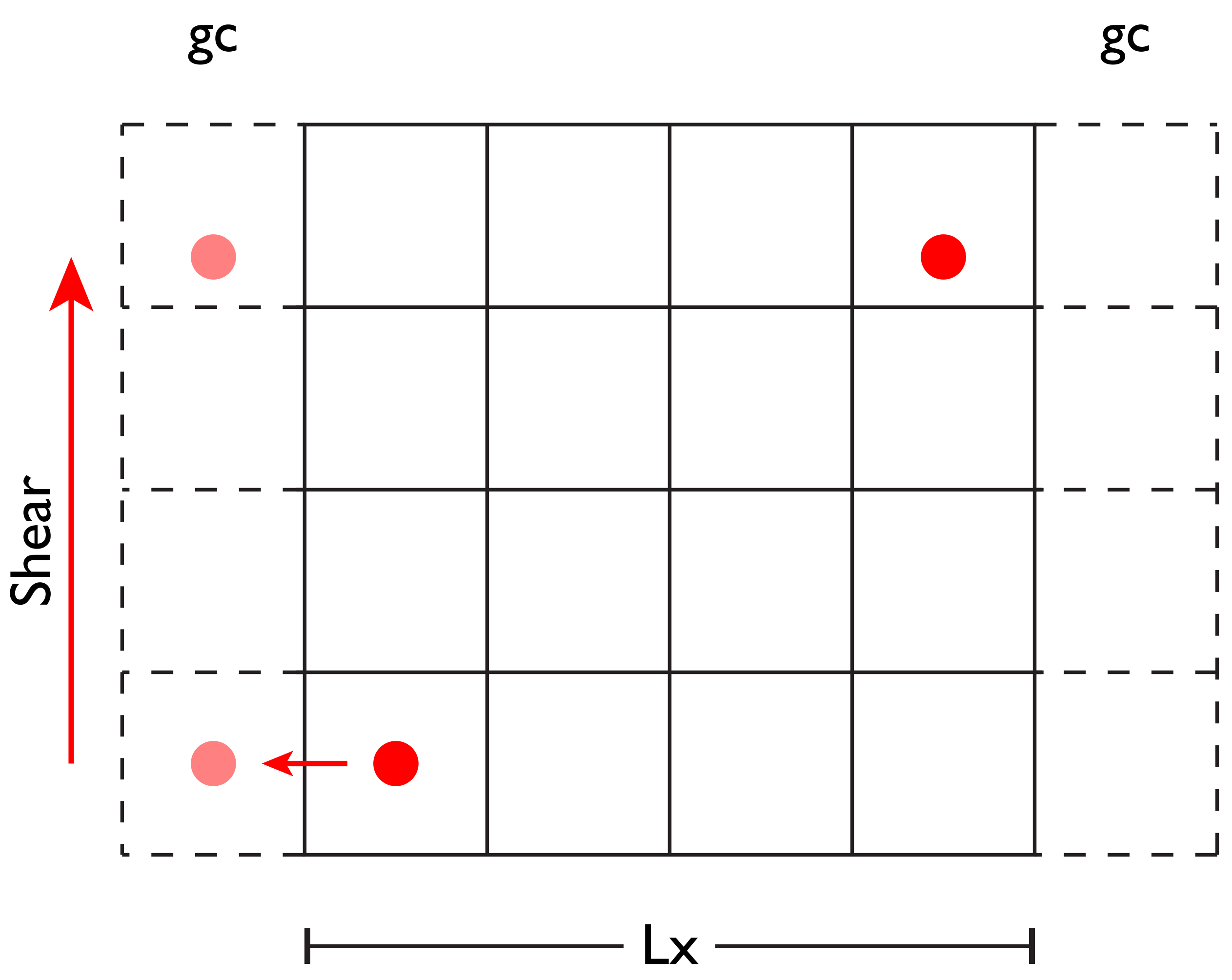
$$Shear(t) = \frac{3}{2}\alpha L_x t$$

s = floor(Shear/dy)

Δ = Shear - s



For grid quantities



For particle

Shear periodic b.c.

$$f(0, y + \frac{3}{2}\alpha L_x t, z) = f(L_x, y, z)$$

for $n_\pm$, B_x, B_y, B_z, J_x, J_z, E_y

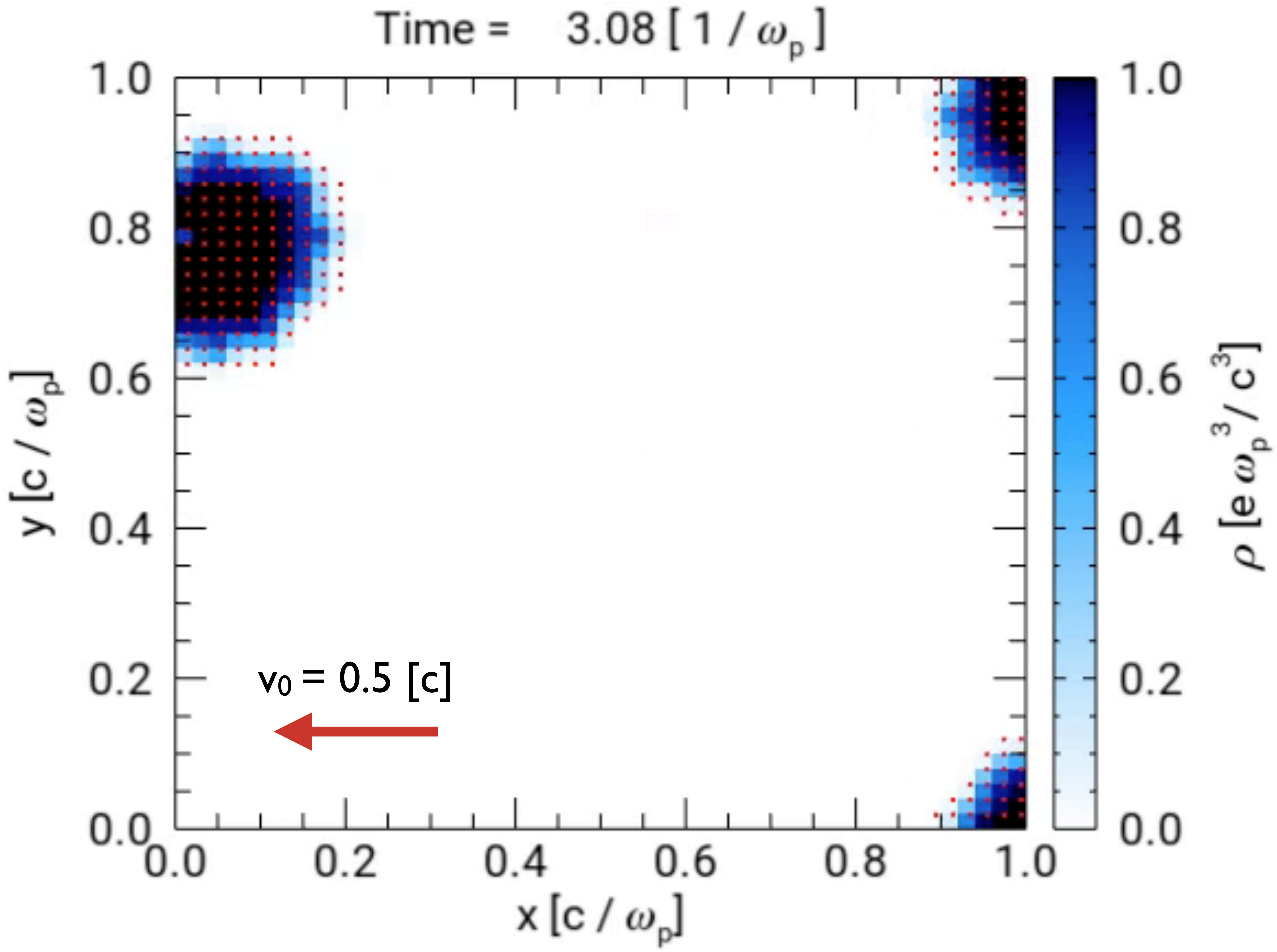
$$E_x(0, y + \frac{3}{2}\alpha L_x t, z) = E_x(L_x, y, z) - \frac{3}{2}\alpha L_x B_z(L_x, y, z)$$

$$E_z(0, y + \frac{3}{2}\alpha L_x t, z) = E_z(L_x, y, z) - \frac{3}{2}\alpha L_x B_x(L_x, y, z)$$

$$J_y^\pm(0, y + \frac{3}{2}\alpha L_x t, z) = J_y^\pm(L_x, y, z) \pm \frac{3}{2}e\alpha L_x n_\pm(L_x, y, z)$$

Benchmark #1

Uniform motion of particles along x



- **Particles**

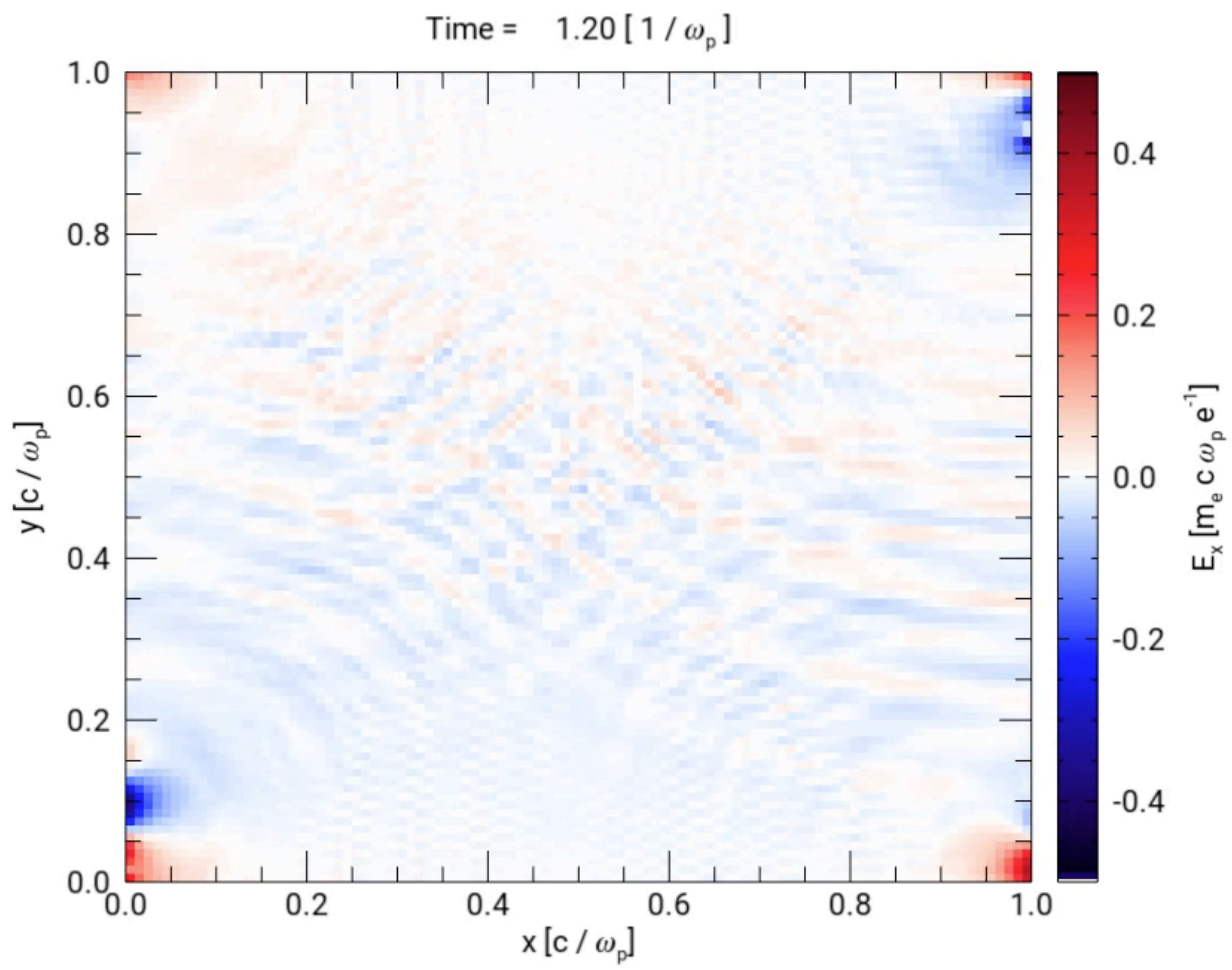
L_x = 1.0 [c/ω_p]

α = 0.05 [1/ω_p]

$$f(0, y + \frac{3}{2}\alpha L_x t, z) = f(L_x, y, z)$$

Benchmark #2

E_x - no plasma, initial E_y field $E_y(x) = \cos(2\pi x)$



The shear periodic b.c introduces a numerical es wave **δE_{error}** at the boundary of E!

$\delta E_{error}/E_0 \sim t/\alpha$

Possible solution: correct E at the boundary using modified poisson equation:

$$\Delta \cdot \left(\vec{E}^{n+1} - \frac{\alpha x}{c} \hat{y} \times \vec{B}^{n+1} \right) - 4\pi\rho^{n+1} = 0$$

Opened to suggestion how to implement it in OSIRIS 4.0 (kyle's **boris correction module**)…

email me for collaboration
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Conclusions & future works

Shearing modules in OSIRIS 4.0
- study of collisionless astrophysical scenarios (accretion disks)
- study of pressure anisotropy instabilities (firehose, mirror, whistler)

2D shearing co-rotating module
- modified Maxwell's and motion equations to include shear terms
- fully tested with analytical 1D MRI linear dispersion relation

3D co-rotating framework + shearing periodic b.c. module
- exceeds the limits of shearing co-rotating framework (no charge conservation in 3D)
- numerical ES wave generated at the x boundary with $\delta E_{error}/E_0 \sim t/\alpha$
- need to implement a Poisson correction for E (to be implemented)

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