Integrating UPIC Algorithms into OSIRIS - Introduction to Spectral Methods in PIC codes

OSIRIS Workshop 2017

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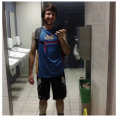
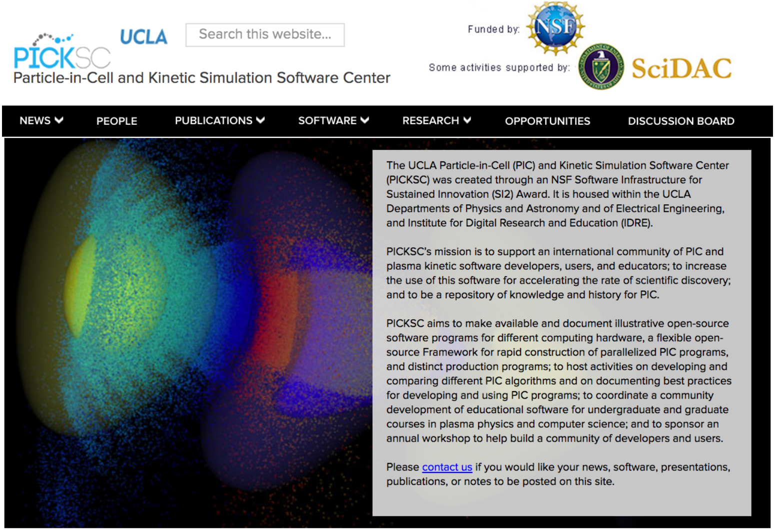


Outline

UPIC framework @ https://picksc.idre.ucla.edu/



UPIC-EMMA (1)

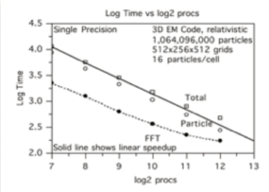
UPIC-EMMA

UPIC-EMMA is a fully relativistic and fully parallelized electromagnetic PIC code, inherited from the UPIC framework[1]

UPIC Features:

- Message-Passing Interface and Multi-Tasking Open-MP parallelization
- Relativistic Binary Collision Module
- Spectral (parallelized FFT algorithm) or
  - Arbitrary order Finite Difference Scheme for solving Maxwell equations
  - Customized solver to eliminate NCI
  - Charge conserving current deposition schemes
- PML absorbing Boundary Conditions + open b.c. for particles
- Parallel I/O for diagnostics (using HDF5, and leverages OSIRIS' visualization packages)
- Antena-generated laser pulses
- High order particle shapes
- Boosted frame
- **GPU & Phi-enabled versions can be adapted quickly from UPIC**

Excellent parallel scalability on supercomputer :
(Strong scaling data, courtesy of V. K. Decyk)



IDEAL for accurate and extensive Laser Plasma interaction simulations related to High Energy Density Physics or Laser-Based Particle Accelerator Physics

PICKSC

http://picksc.idre.ucla.edu/

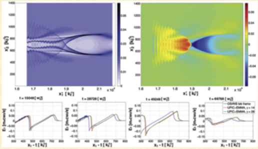
3

[1] V. K. Decyk, Comp. Phys. Com., Volume 177, Issues 1–2, July 2007, Pages 95-97

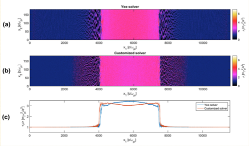
UPIC-EMMA (2)

UPIC-EMMA

Some UPIC-EMMA Applications



Laser wakefield simulation in the Lorentz boosted frame[2]



Relativistic shock simulation[3]

UPIC-EMMA will soon be available on GitHub through an Open Source License:

GitHub Group ID: **UCLA-Plasma-Simulation-Group**
Project ID: **upic-emma**

http://picksc.idre.ucla.edu/

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[2] P.Yu et al., Jour. Comp. Phys., Volume 266, 1 June 2014, Pages 124-138
[3] P.Yu, Yu, PhD Dissertation, UCLA: Electrical Engineering 0303 (2016)

4

Units

Units

Let's note the plasma electron frequency and the thermal electron velocity

$$\omega_p = \sqrt{\frac{4\pi n_{e,0} e^2}{m_e}} \text{ where } n_{e,0} = \frac{N_e d_e}{N_x N_y N_z \Delta^3} \frac{1}{\underline{\Delta}_x \underline{\Delta}_y \underline{\Delta}_z} \text{ and } v_{Th} = \sqrt{\frac{k_B T_{e,0}}{m_e}}.$$

Normalized quantity	$\underline{t}$	$\underline{\mathbf{r}}$	$\underline{\mathbf{k}}$	$\underline{\mathbf{v}}_{a,\ell}$	$\underline{m}_a$	$\underline{\mathbf{p}}_{a,\ell}$	$\underline{q}_a$	$\underline{\rho}$
$=$	ωt	$\frac{\mathbf{r}}{\Delta}$	$\mathbf{k}\Delta$	$\frac{\mathbf{v}_{a,\ell}}{\omega\Delta}$	$\frac{m_a}{m_e}$	$\frac{\mathbf{p}_{a,\ell}}{m_a\omega\Delta}$	$\frac{q_a}{e}$	$\frac{\rho}{en_0}$

Normalized quantity	$\underline{\mathbf{j}}$	$\underline{\mathbf{E}}$	$\underline{\mathbf{B}}$
$=$	$\frac{\mathbf{j}}{en_0\Delta\omega}$	$\frac{e\mathbf{E}}{m_e\omega^2\Delta}$	$\frac{e\mathbf{B}}{m_e\omega^2\Delta}$

$(\Delta,\ \omega)$ are free parameters choosen by the user :

$$\left\{\begin{array}{llcl} \text{Electrostatic units :} & (\Delta,\ \omega) & = & (\ v_{Th}/\omega_p\ ,\ \omega_p\ ) \\ \text{Electromagnetic units :} & (\Delta,\ \omega) & = & (\ c/\omega_p\ ,\ \omega_p\ ) \\ \text{LPI units :} & (\Delta,\ \omega) & = & (\ c/\omega_L\ ,\ \omega_L\ ) \end{array}\right.$$

OSIRIS versus UPIC-EMMA units

Electrical Charge and Current Deposit

$$\underline{\rho} = \sum_{a=e,\,i} \sum_{\ell=1}^{N_a} \underline{q}_a w_f S_a^3 \left[\underline{\mathbf{r}} - \underline{\mathbf{r}}_{a,\ell}(\underline{t})\right] \text{ and } \underline{\mathbf{j}} = \sum_{a=e,\,i} \sum_{\ell=1}^{N_a} \underline{q}_a w_f \underline{\mathbf{v}}_{a,\ell}(\underline{t})\; S_a^3 \left[\underline{\mathbf{r}} - \underline{\mathbf{r}}_{a,\ell}(\underline{t})\right] \text{ where } w_f = \frac{1}{n_0 \Delta^3}$$

Maxwell Equations

Maxwell-Gauss : $\underline{\nabla} \cdot \underline{\mathbf{E}} = a_f\, \underline{\rho}$ Maxwell-Faraday : $\frac{\partial \underline{\mathbf{B}}}{\partial \underline{t}} = -\, \underline{c}\, \underline{\nabla} \times \underline{\mathbf{E}}$

Maxwell-Thomson : $\underline{\nabla} \cdot \underline{\mathbf{B}} = 0$ Maxwell-Ampere : $\frac{\partial \underline{\mathbf{E}}}{\partial \underline{t}} = \underline{c}\, \underline{\nabla} \times \underline{\mathbf{B}} - a_f \underline{\mathbf{j}}$

where $\underline{c} = \frac{c}{\omega \Delta}$ and $a_f = \frac{{\omega_0}^2}{\omega^2}$ with $\omega_0 = \sqrt{\frac{4\pi n_0 e^2}{m_e}}$

Macro particle pusher

$$\frac{d\underline{\mathbf{r}}_{a,\ell}}{d\underline{t}} = \underline{\mathbf{v}}_{a,\ell}(\underline{t}) = \frac{\underline{\mathbf{p}}_{a,\ell}(\underline{t})}{\gamma_{a,\ell}(\underline{t})} \quad \frac{d\underline{\mathbf{p}}_{a,\ell}}{d\underline{t}} = \frac{\underline{q}_a}{\underline{m}_a}\left(\underline{\mathbf{E}} + \frac{\underline{\mathbf{v}}_{a,\ell}}{\underline{c}} \times \underline{\mathbf{B}}\right)$$

OSIRIS choice

$$a_f = 1 \Leftrightarrow n_0 = \frac{m_e \omega^2}{4\pi e^2} \text{ and } w_f = \frac{1}{n_0 \Delta^3}$$

UPIC-EMMA choice

$$w_f = 1 \Leftrightarrow n_0 = \frac{1}{\Delta^3} \text{ and } a_f = \frac{4\pi e^2}{m_e \omega^2 \Delta^3}$$

Longitudinal and Transverse components Splitting

Longitudinal and Transverse components Splitting

Let us note

$$\underline{\mathbf{E}} = \underline{\mathbf{E}}_L + \underline{\mathbf{E}}_T \text{ and } \underline{\mathbf{j}} = \underline{\mathbf{j}}_L + \underline{\mathbf{j}}_T$$

$$\text{where } \begin{cases} \underline{\nabla} \cdot \underline{\mathbf{E}}_T = 0 \\ \underline{\nabla} \times \underline{\mathbf{E}}_L = \mathbf{0} \end{cases} \text{ and } \begin{cases} \underline{\nabla} \cdot \underline{\mathbf{j}}_T = 0 \\ \underline{\nabla} \times \underline{\mathbf{j}}_L = \mathbf{0} \end{cases}$$

Maxwell equations with UPIC-EMMA units

Maxwell-Gauss : $\underline{\nabla} \cdot \underline{\mathbf{E}}_L = a_f\, \underline{\rho}$ and $\underline{\nabla} \cdot \underline{\mathbf{E}}_T = 0$

Maxwell-Thomson : $\underline{\nabla} \cdot \underline{\mathbf{B}} = 0$

Maxwell-Faraday : $\dfrac{\partial \underline{\mathbf{B}}}{\partial \underline{t}} = -\, \underline{c}\, \underline{\nabla} \times \underline{\mathbf{E}}_T$ and $\underline{\nabla} \times \underline{\mathbf{E}}_L = \mathbf{0}$

Maxwell-Ampere : $\dfrac{\partial \underline{\mathbf{E}}_T}{\partial \underline{t}} = \underline{c}\, \underline{\nabla} \times \underline{\mathbf{B}} - a_f \underline{\mathbf{j}}_T$ and $\dfrac{\partial \underline{\mathbf{E}}_L}{\partial \underline{t}} = -a_f \underline{\mathbf{j}}_L$

Outline

Spectral Method (1)

Continuous and $\underline{L}_x$, $\underline{L}_y$ and $\underline{L}_z$-Periodic Solutions : Fourier series

$\forall \underline{F} \in \left\{\underline{\rho},\, \underline{\mathbf{j}},\, \underline{\mathbf{E}},\, \underline{\mathbf{B}}\right\}$

$$\underline{F}\left(\underline{\mathbf{r}},\, \underline{t}\right) = \sum_{p=-\infty}^{\infty} \sum_{q=-\infty}^{\infty} \sum_{r=-\infty}^{\infty} \widehat{\underline{F}}^{p,q,r}\left(\underline{t}\right) \exp\left[\iota\left(\underline{k}_x^p \underline{x} + \underline{k}_y^q \underline{y} + \underline{k}_z^r \underline{z}\right)\right]$$

where

$$\widehat{\underline{F}}^{p,q,r}\left(\underline{t}\right) = \frac{1}{\underline{L}_x}\frac{1}{\underline{L}_y}\frac{1}{\underline{L}_z}\int_0^{\underline{L}_x} d\underline{x} \int_0^{\underline{L}_y} d\underline{y} \int_0^{\underline{L}_z} d\underline{z}\, \underline{F}\left(\underline{\mathbf{r}},\, \underline{t}\right) \exp\left[-\iota\left(\underline{k}_x^p \underline{x} + \underline{k}_y^q \underline{y} + \underline{k}_z^r \underline{z}\right)\right]$$

and

$$\forall\left(p,\, q,\, r\right) \in \mathbb{N}^3,\, \underline{\mathbf{k}}_{p,q,r} = \left(\begin{array}{c} \underline{k}_x^p \\ \underline{k}_y^q \\ \underline{k}_z^r \end{array}\right) = \left(\begin{array}{c} 2\pi\left(p-1\right)/\underline{\Delta}_x \\ 2\pi\left(q-1\right)/\underline{\Delta}_y \\ 2\pi\left(r-1\right)/\underline{\Delta}_z \end{array}\right).$$

All Electromagnetic Modes $\left(\mathbf{E}^{p,q,r},\, \mathbf{B}^{p,q,r}\right)$ are given now by Maxwell Equations with...

$$\underline{\nabla} \rightarrow \iota \underline{\mathbf{k}}_{p,q,r}$$

(ideal method for Linear Partial Differential Equations)

Spectral Method (2)

Discretization of space and time

Let us note $N_x = \underline{L}_x / \underline{\Delta}_x$, $N_y = \underline{L}_y / \underline{\Delta}_y$, $N_z = \underline{L}_z / \underline{\Delta}_z$, $\underline{t}_n = (n-1)\,\underline{\Delta}_t$ and

$$\forall\,(i,\,j,\,k) \in [1,\,N_x] \times [1,\,N_y] \times [1,\,N_z]\,,\; \underline{\mathbf{r}}_{i,j,k} = \begin{pmatrix} \underline{x}_i \\ \underline{y}_j \\ \underline{z}_k \end{pmatrix} = \begin{pmatrix} (i-1)\,\underline{\Delta}_x \\ (j-1)\,\underline{\Delta}_y \\ (r-1)\,\underline{\Delta}_z \end{pmatrix}.$$

Discrete and $\underline{L}_x$, $\underline{L}_y$ and $\underline{L}_z$-Periodic Solutions : Discrete Fourier Transform (DFT)

$$\underline{F}^{i,j,k,n} = \underline{F}\left(\underline{\mathbf{r}}_{i,j,k},\,\underline{t}_n\right) \underset{\text{DFT}}{=} \sum_{p=1}^{N_x}\sum_{q=1}^{N_y}\sum_{r=1}^{N_z} \widehat{\underline{F}}^{p,q,r,n} \exp\left[\iota\left(\underline{k}_x^p \underline{x}_i + \underline{k}_y^q \underline{y}_j + \underline{k}_z^r \underline{z}_k\right)\right]$$

$$\text{and } \widehat{\underline{F}}^{p,q,r,n} \underset{\text{IDFT}}{=} \frac{1}{N_x}\frac{1}{N_y}\frac{1}{N_z} \sum_{i=1}^{N_x}\sum_{q=1}^{N_y}\sum_{r=1}^{N_z} \underline{F}^{i,j,k,n} \exp\left[-\iota\left(\underline{k}_x^p \underline{x}_i + \underline{k}_y^q \underline{y}_j + \underline{k}_z^r \underline{z}_k\right)\right]$$

that becomes $2\pi/\underline{\Delta}_x$, $2\pi/\underline{\Delta}_y$ and $2\pi/\underline{\Delta}_z$-periodic along the k_x, k_y and k_z-direction.

$$\Rightarrow \underline{\mathbf{k}}_{\text{Nyquist}} = \left(\underline{k}_x^{1+\frac{N_x}{2}},\, \underline{k}_y^{1+\frac{N_y}{2}},\, \underline{k}_z^{1+\frac{N_z}{2}}\right) = \left(\frac{\pi}{\underline{\Delta}_x},\, \frac{\pi}{\underline{\Delta}_y},\, \frac{\pi}{\underline{\Delta}_z}\right) = \text{highest mode.}$$

Outline

UPIC-EMMA Maxwell Solver (1)

UPIC-EMMA Maxwell Solver

$$\underline{\widehat{\mathbf{E}}}_L^{p,q,r,n+1} = -\frac{\iota \underline{\mathbf{k}}_{p,q,r}}{\underline{\mathbf{k}}_{p,q,r}{}^2}\, a_f\, \underline{\widehat{\rho}}^{p,q,r,n+1}$$

$$\underline{\widehat{\mathbf{j}}}_T^{p,q,r,n+1/2} = \underline{\widehat{\mathbf{j}}}^{p,q,r,n+1/2} - \underline{\widehat{\mathbf{j}}}_L^{p,q,r,n+1/2} \text{ with } \underline{\widehat{\mathbf{j}}}_L^{p,q,r,n+1/2} = \frac{\underline{\mathbf{k}}_{p,q,r}}{\underline{\mathbf{k}}_{p,q,r}{}^2}\underline{\mathbf{k}}_{p,q,r} \cdot \underline{\widehat{\mathbf{j}}}^{p,q,r,n+1/2}$$

$$\begin{cases} \underline{\widehat{\mathbf{B}}}^{p,q,r,n+1/2} = \underline{\widehat{\mathbf{B}}}^{p,q,r,n} - \frac{\Delta_t}{2}\, c \left(\iota \underline{\mathbf{k}}_{p,q,r}\right) \times \underline{\widehat{\mathbf{E}}}_T^{p,q,r,n} \\ \underline{\widehat{\mathbf{E}}}_T^{p,q,r,n+1} = \underline{\widehat{\mathbf{E}}}_T^{p,q,r,n} + \Delta_t\, c \left(\iota \underline{\mathbf{k}}_{p,q,r}\right) \times \underline{\widehat{\mathbf{B}}}^{p,q,r,n+1/2} \\ \qquad - \Delta_t\, a_f\, \underline{\widehat{\mathbf{j}}}_T^{p,q,r,n+1/2} \\ \underline{\widehat{\mathbf{B}}}^{p,q,r,n+1} = \underline{\widehat{\mathbf{B}}}^{p,q,r,n+1/2} - \frac{\Delta_t}{2}\, c \left(\iota \underline{\mathbf{k}}_{p,q,r}\right) \times \underline{\widehat{\mathbf{E}}}_T^{p,q,r,n+1} \end{cases}$$

without forgetting

$$\begin{cases} \iota \mathbf{k}_{p,q,r} \cdot \underline{\mathbf{E}}_T^{p,q,r,n} = 0 \\ \iota \mathbf{k}_{p,q,r} \cdot \underline{\mathbf{j}}_T^{p,q,r,n+\frac{1}{2}} = 0 \\ \iota \mathbf{k}_{p,q,r} \cdot \underline{\mathbf{B}}^{p,q,r,n+\frac{1}{2}} = 0 \end{cases} \Rightarrow \forall (p,\ q,\ r) \neq 1, \begin{cases} \underline{E}_{T,x}^{p,1,1,n} = \underline{E}_{T,y}^{1,q,1,n} = \underline{E}_{T,z}^{1,1,r,n} = 0 \\ \underline{j}_{T,x}^{p,1,1,n+\frac{1}{2}} = \underline{j}_{T,y}^{1,q,1,n+\frac{1}{2}} = \underline{j}_{T,z}^{1,1,r,n+\frac{1}{2}} = 0 \\ \underline{B}_x^{p,1,1,n+\frac{1}{2}} = \underline{B}_y^{1,q,1,n+\frac{1}{2}} = \underline{B}_z^{1,1,r,n+\frac{1}{2}} = 0 \end{cases}.$$

UPIC-EMMA Maxwell Solver (2)

Deduced Numerical Helmoltz Equations

$$\frac{\underline{\hat{\mathbf{E}}}_T^{p,q,r,n+1} - 2\underline{\hat{\mathbf{E}}}_T^{p,q,r,n} + \underline{\hat{\mathbf{E}}}_T^{p,q,r,n-1}}{\underline{\Delta}_t{}^2} + \underline{\mathbf{k}}_{p,q,r}{}^2 \underline{c}^2 \underline{\hat{\mathbf{E}}}_T^{p,q,r,n} = -a_f \frac{\underline{\hat{\mathbf{j}}}_T^{p,q,r,n+\frac{1}{2}} - \underline{\hat{\mathbf{j}}}_T^{p,q,r,n-\frac{1}{2}}}{\underline{\Delta}_t}$$

$$\frac{\underline{\hat{\mathbf{B}}}^{p,q,r,n+\frac{3}{2}} - 2\underline{\hat{\mathbf{B}}}^{p,q,r,n+\frac{1}{2}} + \underline{\hat{\mathbf{B}}}^{p,q,r,n-\frac{1}{2}}}{\underline{\Delta}_t{}^2} + \underline{\mathbf{k}}_{p,q,r}{}^2 \underline{c}^2 \underline{\hat{\mathbf{B}}}_T^{p,q,r,n+\frac{1}{2}} = a_f \underline{c} \left(\iota \underline{\mathbf{k}}_{p,q,r}\right) \times \underline{\hat{\mathbf{j}}}_T^{p,q,r,n+\frac{1}{2}}$$

Deduced Stability Condition

$$\Delta_t \leq \sqrt{2} / \left|\underline{\mathbf{k}}_{\mathrm{Nyquist}}\right| \underline{c}.$$

SLIGHTLY MORE RESTRICTIVE THAN THE YEE SCHEME CFL CONDITION!

UPIC-EMMA Maxwell Solver (3)

Electromagnetic Fields Initialization : Darwin Fields

$$\widehat{\underline{\mathbf{E}}}_L^{p,q,r,0} = -\frac{\iota \underline{\mathbf{k}}_{p,q,r}}{\underline{\mathbf{k}}_{p,q,r}{}^2}\, a_f\, \widehat{\underline{\rho}}^{p,q,r,0},$$

$$\widehat{\underline{\mathbf{j}}}_T^{p,q,r,0} = \widehat{\underline{\mathbf{j}}}^{p,q,r,0} - \widehat{\underline{\mathbf{j}}}_L^{p,q,r,0} \text{ with } \widehat{\underline{\mathbf{j}}}_L^{p,q,r,0} = \frac{\underline{\mathbf{k}}_{p,q,r}}{\underline{\mathbf{k}}_{p,q,r}{}^2}\underline{\mathbf{k}}_{p,q,r} \cdot \widehat{\underline{\mathbf{j}}}^{p,q,r,0},$$

$$\left.\frac{\partial \widehat{\underline{\mathbf{j}}}_T}{\partial t}\right|^{p,q,r,0} = \left.\frac{\partial \widehat{\underline{\mathbf{j}}}}{\partial t}\right|^{p,q,r,0} - \left.\frac{\partial \widehat{\underline{\mathbf{j}}}_L}{\partial t}\right|^{p,q,r,0} \text{ with } \left.\frac{\partial \widehat{\underline{\mathbf{j}}}_L}{\partial t}\right|^{p,q,r,0} = \frac{\underline{\mathbf{k}}_{p,q,r}}{\underline{\mathbf{k}}_{p,q,r}{}^2}\underline{\mathbf{k}}_{p,q,r} \cdot \left.\frac{\partial \widehat{\underline{\mathbf{j}}}}{\partial t}\right|^{p,q,r,0}$$

and

$$\left\{\begin{array}{lll} \widehat{\underline{\mathbf{B}}}^{p,q,r,0} = & \dfrac{a_f}{\underline{c}}\dfrac{1}{\underline{\mathbf{k}}_{p,q,r}{}^2} & \left(\iota \underline{\mathbf{k}}_{p,q,r}\right) \times \widehat{\underline{\mathbf{j}}}_T^{p,q,r,0} \\ \widehat{\underline{\mathbf{E}}}_T^{p,q,r,0} = & -\dfrac{a_f}{\underline{c}^2}\dfrac{1}{\underline{\mathbf{k}}_{p,q,r}{}^2} & \left.\dfrac{\partial \widehat{\underline{\mathbf{j}}}_T}{\partial t}\right|^{p,q,r,0} \end{array}\right. \ll \left\{\begin{array}{lll} \underline{\nabla} \times \underline{\mathbf{B}} & = \dfrac{a_f}{\underline{c}}\widehat{\underline{\mathbf{j}}} & + \dfrac{1}{\underline{c}}\dfrac{\partial \widehat{\underline{\mathbf{E}}}_L}{\partial \underline{t}} \\ \underline{\nabla} \times \underline{\mathbf{E}}_T & = & -\dfrac{1}{\underline{c}}\dfrac{\partial \widehat{\underline{\mathbf{B}}}}{\partial \underline{t}} \end{array}\right.$$

without forgetting

$$\forall (p,\ q,\ r) \neq 1,\ \left\{\begin{array}{l} \underline{E}_{T,x}^{p,1,1,0} = \underline{E}_{T,y}^{1,q,1,0} = \underline{E}_{T,z}^{1,1,r,0} = 0 \\ \underline{B}_x^{p,1,1,0} = \underline{B}_y^{1,q,1,0} = \underline{B}_z^{1,1,r,0} = 0 \end{array}\right. .$$

Outline

Yee Scheme with an order of acuracy $N = 2s$ - Magnetic Field Update

Yee Scheme with an order of acuracy $N = 2s$ - Magnetic Field Update

$$
\left\{
\begin{aligned}
\frac{\underline{B}_x^{i,j-\frac{1}{2},k-\frac{1}{2},n+\frac{1}{2}} - \underline{B}_x^{i,j-\frac{1}{2},k-\frac{1}{2},n-\frac{1}{2}}}{\underline{\Delta}_t} &= \underline{c}\sum_{m=1}^{s} C_m \frac{\underline{E}_{T,y}^{i,j-\frac{1}{2},k+m-1,n} - \underline{E}_{T,y}^{i,j-\frac{1}{2},k-m,n}}{\underline{\Delta}_z} \\
&- \underline{c}\sum_{m=1}^{s} C_m \frac{\underline{E}_{T,z}^{i,j+m-1,k-\frac{1}{2},n} - \underline{E}_{T,z}^{i,j-m,k-\frac{1}{2},n}}{\underline{\Delta}_y} \\
\frac{\underline{B}_y^{i-\frac{1}{2},j,k-\frac{1}{2},n+\frac{1}{2}} - \underline{B}_y^{i-\frac{1}{2},j,k-\frac{1}{2},n-\frac{1}{2}}}{\underline{\Delta}_t} &= \underline{c}\sum_{m=1}^{s} C_m \frac{\underline{E}_{T,z}^{i+m-1,j,k-\frac{1}{2},n} - \underline{E}_{T,z}^{i-m,j,k-\frac{1}{2},n}}{\underline{\Delta}_x} \\
&- \underline{c}\sum_{m=1}^{s} C_m \frac{\underline{E}_{T,x}^{i-\frac{1}{2},j,k+m-1,n} - \underline{E}_{T,x}^{i-\frac{1}{2},j,k-m,n}}{\underline{\Delta}_z} \\
\frac{\underline{B}_z^{i-\frac{1}{2},j-\frac{1}{2},k,n+\frac{1}{2}} - \underline{B}_z^{i-\frac{1}{2},j-\frac{1}{2},k,n-\frac{1}{2}}}{\underline{\Delta}_t} &= \underline{c}\sum_{m=1}^{s} C_m \frac{\underline{E}_{T,x}^{i-\frac{1}{2},j+m-1,k,n} - \underline{E}_{T,x}^{i-\frac{1}{2},j-m,k,n}}{\underline{\Delta}_y} \\
&- \underline{c}\sum_{m=1}^{s} C_m \frac{\underline{E}_{T,y}^{i+m-1,j-\frac{1}{2},k,n} - \underline{E}_{T,y}^{i-m,j-\frac{1}{2},k,n}}{\underline{\Delta}_x}
\end{aligned}
\right.
$$

with

$$
C_m = \frac{1}{2m-1} \prod_{\substack{l=1 \\ l \neq m}}^{s} \frac{(2l-1)^2}{(2l-1)^2 - (2m-1)^2}.
$$

Yee Scheme with an order of acuracy $N = 2s$ - Electric Field Update

Yee Scheme with an order of acuracy $N = 2s$ - Electric Field Update

$$
\left\{
\begin{aligned}
\frac{\underline{E}_{T,x}^{i-\frac{1}{2},j,k,n+1} - \underline{E}_{T,x}^{i-\frac{1}{2},j,k,n}}{\underline{\Delta}_t} &= -a_f \underline{j}_{T,x}^{i-\frac{1}{2},j,k,n+\frac{1}{2}} \\
&+ \underline{c} \sum_{m=1}^{s} C_m \frac{\underline{B}_z^{i-\frac{1}{2},j-\frac{1}{2}+m,k,n+\frac{1}{2}} - \underline{B}_z^{i-\frac{1}{2},j-\frac{1}{2}-m+1,k,n+\frac{1}{2}}}{\underline{\Delta}_y} \\
&- \underline{c} \sum_{m=1}^{s} C_m \frac{\underline{B}_y^{i-\frac{1}{2},j,k-\frac{1}{2}+m,n+\frac{1}{2}} - \underline{B}_y^{i-\frac{1}{2},j,k-\frac{1}{2}-m+1,n+\frac{1}{2}}}{\underline{\Delta}_z} \\
\frac{\underline{E}_{T,y}^{i,j-\frac{1}{2},k,n+1} - \underline{E}_{T,y}^{i,j-\frac{1}{2},k,n}}{\underline{\Delta}_t} &= -a_f \underline{j}_{T,y}^{i,j-\frac{1}{2},k,n+\frac{1}{2}} \\
&= \underline{c} \sum_{m=1}^{s} C_m \frac{\underline{B}_x^{i,j-\frac{1}{2},k-\frac{1}{2}+m,n+\frac{1}{2}} - \underline{B}_x^{i,j-\frac{1}{2},k-\frac{1}{2}-m+1,n+\frac{1}{2}}}{\underline{\Delta}_z} \\
&- \underline{c} \sum_{m=1}^{s} C_m \frac{\underline{B}_z^{i-\frac{1}{2}+m,j-\frac{1}{2},k,n-\frac{1}{2}} - \underline{B}_z^{i-\frac{1}{2}-m+1,j-\frac{1}{2},k,n+\frac{1}{2}}}{\underline{\Delta}_x} \\
\frac{\underline{E}_{T,z}^{i,j,k-\frac{1}{2},n+1} - \underline{E}_{T,z}^{i,j,k-\frac{1}{2},n}}{\underline{\Delta}_t} &= -a_f \underline{j}_{T,z}^{i,j,k-\frac{1}{2},n+\frac{1}{2}} \\
&= \underline{c} \sum_{m=1}^{s} C_m \frac{\underline{B}_y^{i-\frac{1}{2}+m,j,k-\frac{1}{2},n+\frac{1}{2}} - \underline{B}_y^{i-\frac{1}{2}-m+1,j,k-\frac{1}{2},n+\frac{1}{2}}}{\underline{\Delta}_x} \\
&- \underline{c} \sum_{m=1}^{s} C_m \frac{\underline{B}_x^{i,j-\frac{1}{2}+m,k-\frac{1}{2},n+\frac{1}{2}} - \underline{B}_x^{i,j-\frac{1}{2}-m+1,k-\frac{1}{2},n+\frac{1}{2}}}{\underline{\Delta}_y}
\end{aligned}
\right.
$$

Outline

Implementation in a Spectral Solver (1)

Inverse Discrete Fourier Transform (IDFT) of B field equations

$$\left\{\begin{array}{lcl}
\dfrac{\left[\widehat{\underline{B}}_x^{p,q,r,n+\frac{1}{2}}\, e^{-\iota\left(\theta_y^q+\theta_z^r\right)}\right]-\left[\widehat{\underline{B}}_x^{p,q,r,n-\frac{1}{2}}\, e^{-\iota\left(\theta_y^q+\theta_z^r\right)}\right]}{\underline{\Delta}_t} & = & \iota\left\{\underline{k}_z^r\right\}\underline{c}\, e^{-\iota\theta_z^r}\left[\widehat{\underline{E}}_{T,y}^{p,q,r,n}\, e^{-\iota\theta_y^q}\right] \\
 & - & \iota\left\{\underline{k}_y^q\right\}\underline{c}\, e^{-\iota\theta_y^q}\left[\underline{E}_{T,z}^{p,q,r,n}\, e^{-\iota\theta_z^r}\right] \\
\dfrac{\left[\widehat{\underline{B}}_y^{p,q,r,n+\frac{1}{2}}\, e^{-\iota\left(\theta_x^p+\theta_z^r\right)}\right]-\left[\widehat{\underline{B}}_y^{p,q,r,n-\frac{1}{2}}\, e^{-\iota\left(\theta_x^p+\theta_z^r\right)}\right]}{\underline{\Delta}_t} & = & \iota\left\{\underline{k}_x^p\right\}\underline{c}\, e^{-\iota\theta_x^p}\left[\widehat{\underline{E}}_{T,z}^{p,q,r,n}\, e^{-\iota\theta_z^r}\right] \\
 & - & \iota\left\{\underline{k}_z^r\right\}\underline{c}\, e^{-\iota\theta_z^r}\left[\widehat{\underline{E}}_{T,x}^{p,q,r,n}\, e^{-\iota\theta_x^p}\right] \\
\dfrac{\left[\widehat{\underline{B}}_z^{p,q,r,n+\frac{1}{2}}\, e^{-\iota\left(\theta_x^p+\theta_y^q\right)}\right]-\left[\widehat{\underline{B}}_z^{p,q,r,n-\frac{1}{2}}\, e^{-\iota\left(\theta_x^p+\theta_y^q\right)}\right]}{\underline{\Delta}_t} & = & \iota\left\{\underline{k}_y^q\right\}\underline{c}\, e^{-\iota\theta_y^q}\left[\widehat{\underline{E}}_{T,x}^{p,q,r,n}\, e^{-\iota\theta_x^p}\right] \\
 & - & \iota\left\{\underline{k}_x^p\right\}\underline{c}\, e^{-\iota\theta_x^p}\left[\widehat{\underline{E}}_{T,y}^{p,q,r,n}\, e^{-\iota\theta_y^q}\right]
\end{array}\right.$$

with
$$\left\{\begin{array}{l}
\left\{\underline{k}_x^p\right\} = \displaystyle\sum_{m=1}^{s} C_m \frac{\sin\left[(2m-1)\,\underline{k}_x^p\underline{\Delta}_x/2\right]}{\underline{\Delta}_x/2} \\
\left\{\underline{k}_y^q\right\} = \displaystyle\sum_{m=1}^{s} C_m \frac{\sin\left[(2m-1)\,\underline{k}_y^q\underline{\Delta}_y/2\right]}{\underline{\Delta}_y/2} \\
\left\{\underline{k}_z^r\right\} = \displaystyle\sum_{m=1}^{s} C_m \frac{\sin\left[(2m-1)\,\underline{k}_z^r\underline{\Delta}_z/2\right]}{\underline{\Delta}_z/2}
\end{array}\right.
\quad\text{and}\quad
\left\{\begin{array}{l}
\theta_x^p = \underline{k}_x^p\underline{\Delta}_x/2 \\
\theta_y^q = \underline{k}_y^q\underline{\Delta}_y/2 \\
\theta_z^r = \underline{k}_z^r\underline{\Delta}_z/2
\end{array}\right. .$$

Implementation in a Spectral Solver (1)

Inverse Discrete Fourier Transform (IDFT) of B field equations

$$
\begin{cases}
\dfrac{\left[\underline{\hat{B}}_x^{p,q,r,n+\frac{1}{2}}\cancel{e^{-\iota\left(\theta_y^q+\theta_z^r\right)}}\right]-\left[\underline{\hat{B}}_x^{p,q,r,n-\frac{1}{2}}\cancel{e^{-\iota\left(\theta_y^q+\theta_z^r\right)}}\right]}{\underline{\Delta}_t} = \iota\left\{\underline{k}_z^r\right\}c\,\cancel{e^{-\iota\theta_z^r}}\left[\underline{\hat{E}}_{T,y}^{p,q,r,n}\cancel{e^{-\iota\theta_y^q}}\right] - \iota\left\{\underline{k}_y^q\right\}c\,\cancel{e^{-\iota\theta_y^q}}\left[\underline{\hat{E}}_{T,z}^{p,q,r,n}\cancel{e^{-\iota\theta_z^r}}\right]\\[2ex]
\dfrac{\left[\underline{\hat{B}}_y^{p,q,r,n+\frac{1}{2}}\cancel{e^{-\iota\left(\theta_x^p+\theta_z^r\right)}}\right]-\left[\underline{\hat{B}}_y^{p,q,r,n-\frac{1}{2}}\cancel{e^{-\iota\left(\theta_x^p+\theta_z^r\right)}}\right]}{\underline{\Delta}_t} = \iota\left\{\underline{k}_x^p\right\}c\,\cancel{e^{-\iota\theta_x^p}}\left[\underline{\hat{E}}_{T,z}^{p,q,r,n}\cancel{e^{-\iota\theta_z^r}}\right] - \iota\left\{\underline{k}_z^r\right\}c\,\cancel{e^{-\iota\theta_z^r}}\left[\underline{\hat{E}}_{T,x}^{p,q,r,n}\cancel{e^{-\iota\theta_x^p}}\right]\\[2ex]
\dfrac{\left[\underline{\hat{B}}_z^{p,q,r,n+\frac{1}{2}}\cancel{e^{-\iota\left(\theta_x^p+\theta_y^q\right)}}\right]-\left[\underline{\hat{B}}_z^{p,q,r,n-\frac{1}{2}}\cancel{e^{-\iota\left(\theta_x^p+\theta_y^q\right)}}\right]}{\underline{\Delta}_t} = \iota\left\{\underline{k}_y^q\right\}c\,\cancel{e^{-\iota\theta_y^q}}\left[\underline{\hat{E}}_{T,x}^{p,q,r,n}\cancel{e^{-\iota\theta_x^p}}\right] - \iota\left\{\underline{k}_x^p\right\}c\,\cancel{e^{-\iota\theta_x^p}}\left[\underline{\hat{E}}_{T,y}^{p,q,r,n}\cancel{e^{-\iota\theta_y^q}}\right]
\end{cases}
$$

$$
\text{where}\quad
\begin{cases}
\left\{\underline{k}_x^p\right\} = \sum_{m=1}^{s} C_m \dfrac{\sin\left[(2m-1)\,\underline{k}_x^p\underline{\Delta}_x/2\right]}{\underline{\Delta}_x/2}\\[2ex]
\left\{\underline{k}_y^q\right\} = \sum_{m=1}^{s} C_m \dfrac{\sin\left[(2m-1)\,\underline{k}_y^q\underline{\Delta}_y/2\right]}{\underline{\Delta}_y/2}\\[2ex]
\left\{\underline{k}_z^r\right\} = \sum_{m=1}^{s} C_m \dfrac{\sin\left[(2m-1)\,\underline{k}_z^r\underline{\Delta}_z/2\right]}{\underline{\Delta}_z/2}
\end{cases}
\quad\text{and}\quad
\begin{cases}
\theta_x^p = \underline{k}_x^p\underline{\Delta}_x/2\\
\theta_y^q = \underline{k}_y^q\underline{\Delta}_y/2\\
\theta_z^r = \underline{k}_z^r\underline{\Delta}_z/2
\end{cases}\ .
$$

Implementation in a Spectral Solver (2)

Inverse Discrete Fourier Transform (IDFT) of E field equations

$$
\begin{cases}
\dfrac{\left[\underline{\widehat{E}}_{T,x}^{p,q,r,n+1}\, e^{-\iota\theta_x^p}\right] - \left[\underline{\widehat{E}}_{T,x}^{p,q,r,n}\, e^{-\iota\theta_x^p}\right]}{\underline{\Delta}_t} & = \;\; -a_f \left[\underline{\widehat{j}}_{T,x}^{p,q,r,n+\frac{1}{2}}\, e^{-\iota\theta_x^p}\right] \\
& - \;\; \iota \left\{\underline{k}_z^r\right\} \underline{c}\, e^{\iota\theta_z^r} \left[\underline{\widehat{B}}_y^{p,q,r,n+\frac{1}{2}}\, e^{-\iota\left(\theta_x^p+\theta_z^r\right)}\right] \\
& + \;\; \iota \left\{\underline{k}_y^q\right\} \underline{c}\, e^{\iota\theta_y^q} \left[\underline{\widehat{B}}_z^{p,q,r,n+\frac{1}{2}}\, e^{-\iota\left(\theta_x^p+\theta_y^q\right)}\right] \\
\dfrac{\left[\underline{\widehat{E}}_{T,y}^{p,q,r,n+1}\, e^{-\iota\theta_y^q}\right] - \left[\underline{\widehat{E}}_{T,y}^{p,q,r,n}\, e^{-\iota\theta_y^q}\right]}{\underline{\Delta}_t} & = \;\; -a_f \left[\underline{\widehat{j}}_{T,y}^{p,q,r,n+\frac{1}{2}}\, e^{-\iota\theta_y^q}\right] \\
& - \;\; \iota \left\{\underline{k}_x^p\right\} \underline{c}\, e^{\iota\theta_x^p} \left[\underline{\widehat{B}}_z^{p,q,r,n+\frac{1}{2}}\, e^{-\iota\left(\theta_x^p+\theta_y^q\right)}\right] \\
& + \;\; \iota \left\{\underline{k}_z^r\right\} \underline{c}\, e^{\iota\theta_z^r} \left[\underline{\widehat{B}}_x^{p,q,r,n+\frac{1}{2}}\, e^{-\iota\left(\theta_y^q+\theta_z^r\right)}\right] \\
\dfrac{\left[\underline{\widehat{E}}_{T,z}^{p,q,r,n+1}\, e^{-\iota\theta_z^r}\right] - \left[\underline{\widehat{E}}_{T,z}^{p,q,r,n}\, e^{-\iota\theta_z^r}\right]}{\underline{\Delta}_t} & = \;\; -a_f \left[\underline{\widehat{j}}_{T,z}^{p,q,r,n+\frac{1}{2}}\, e^{-\iota\theta_z^r}\right] \\
& - \;\; \iota \left\{\underline{k}_y^q\right\} \underline{c}\, e^{\iota\theta_y^q} \left[\underline{\widehat{B}}_x^{p,q,r,n+\frac{1}{2}}\, e^{-\iota\left(\theta_y^q+\theta_z^r\right)}\right] \\
& + \;\; \iota \left\{\underline{k}_x^p\right\} e^{\iota\theta_x^p}\, \underline{c} \left[\underline{\widehat{B}}_y^{p,q,r,n+\frac{1}{2}}\, e^{-\iota\left(\theta_x^p+\theta_z^r\right)}\right]
\end{cases} .
$$

Implementation in a Spectral Solver (2)

Inverse Discrete Fourier Transform (IDFT) of E field equations

$$
\left\{
\begin{aligned}
\frac{\left[\widehat{\underline{E}}_{T,x}^{p,q,r,n+1}\,\cancel{e^{-\iota\theta_x^p}}\right]-\left[\widehat{\underline{E}}_{T,x}^{p,q,r,n}\,\cancel{e^{-\iota\theta_x^p}}\right]}{\underline{\Delta}_t}
&= -a_f\left[\widehat{\underline{j}}_{T,x}^{p,q,r,n+\frac{1}{2}}\,\cancel{e^{-\iota\theta_x^p}}\right]\\
&\quad-\iota\left\{\underline{k}_z^r\right\}\underline{c}\,\cancel{e^{\iota\theta_z^r}}\left[\widehat{\underline{B}}_y^{p,q,r,n+\frac{1}{2}}\,\cancel{e^{-\iota(\theta_x^p+\theta_z^r)}}\right]\\
&\quad+\iota\left\{\underline{k}_y^q\right\}\underline{c}\,\cancel{e^{\iota\theta_y^q}}\left[\underline{B}_z^{p,q,r,n+\frac{1}{2}}\,\cancel{e^{-\iota(\theta_x^p+\theta_y^q)}}\right]\\
\frac{\left[\widehat{\underline{E}}_{T,y}^{p,q,r,n+1}\,\cancel{e^{-\iota\theta_y^q}}\right]-\left[\widehat{\underline{E}}_{T,y}^{p,q,r,n}\,\cancel{e^{-\iota\theta_y^q}}\right]}{\underline{\Delta}_t}
&= -a_f\left[\widehat{\underline{j}}_{T,y}^{p,q,r,n+\frac{1}{2}}\,\cancel{e^{-\iota\theta_y^q}}\right]\\
&\quad-\iota\left\{\underline{k}_x^p\right\}\underline{c}\,\cancel{e^{\iota\theta_x^p}}\left[\widehat{\underline{B}}_z^{p,q,r,n+\frac{1}{2}}\,\cancel{e^{-\iota(\theta_x^p+\theta_y^q)}}\right]\\
&\quad+\iota\left\{\underline{k}_z^r\right\}\underline{c}\,\cancel{e^{\iota\theta_z^r}}\left[\widehat{\underline{B}}_x^{p,q,r,n+\frac{1}{2}}\,\cancel{e^{-\iota(\theta_y^q+\theta_z^r)}}\right]\\
\frac{\left[\widehat{\underline{E}}_{T,z}^{p,q,r,n+1}\,\cancel{e^{-\iota\theta_z^r}}\right]-\left[\widehat{\underline{E}}_{T,z}^{p,q,r,n}\,\cancel{e^{-\iota\theta_z^r}}\right]}{\underline{\Delta}_t}
&= -a_f\left[\widehat{\underline{j}}_{T,z}^{p,q,r,n+\frac{1}{2}}\,\cancel{e^{-\iota\theta_z^r}}\right]\\
&\quad-\iota\left\{\underline{k}_y^q\right\}\underline{c}\,\cancel{e^{\iota\theta_y^q}}\left[\widehat{\underline{B}}_x^{p,q,r,n+\frac{1}{2}}\,\cancel{e^{-\iota(\theta_y^q+\theta_z^r)}}\right]\\
&\quad+\iota\left\{\underline{k}_x^p\right\}\cancel{e^{\iota\theta_x^p}}\,\underline{c}\left[\widehat{\underline{B}}_y^{p,q,r,n+\frac{1}{2}}\,\cancel{e^{-\iota(\theta_x^p+\theta_z^r)}}\right]
\end{aligned}
\right. .
$$

Outline

Initialization

Nominal Simulation Parameters

- 2D
- Periodic boundary conditions
- Electromagnetic units ($\underline{c} = 1$)
- 4 MPI nodes and 2 OpenMP threads per node
- $\underline{L}_x = \underline{L}_y = 128$
- $\underline{L}_{\mathrm{PML}} = 30$
- $\underline{\Delta}_x = \underline{\Delta}_y = 1$
- $\underline{L}_t = 128$
- $a_f = 1$

Electrical Current Density Initialization and Expected Solutions

$$\underline{\mathbf{j}}_T^{i,j,k,n} = \begin{cases} \underline{j}_0 \sin\left(\underline{\omega}_0 \underline{t}_n\right) \mathbf{e}_z & \text{if } \begin{cases} 63 \leq \underline{x}_i \leq 65 \\ 63 \leq \underline{y}_j \leq 65 \end{cases} \\ 0 & \text{else} \end{cases} \qquad \text{with } \underline{\omega}_0 = 1 \text{ and } \underline{j}_0 = -10.$$

$$\Rightarrow \begin{cases} \underline{E}_{T,r}^{i,j,k,n} & = & 0 \\ \underline{E}_{T,z}^{i,j,k,n} & \underset{\left|\underline{\mathbf{r}}_{i,j,k} - \underline{\mathbf{r}}_0\right| \gg 1}{\approx} & \dfrac{\underline{E}_0}{\sqrt{\left|\underline{\mathbf{r}}_{i,j,k} - \underline{\mathbf{r}}_0\right|}} \cos\left(\underline{\omega}_0 \underline{t}_n - \underline{k}_0 \left|\underline{\mathbf{r}}_{i,j,k} - \underline{\mathbf{r}}_0\right|\right) \\ \underline{B}_r^{i,j,k,n} & \underset{\left|\underline{\mathbf{r}}_{i,j,k} - \underline{\mathbf{r}}_0\right| \gg 1}{\approx} & \dfrac{\underline{E}_0}{\sqrt{\left|\underline{\mathbf{r}}_{i,j,k} - \underline{\mathbf{r}}_0\right|}} \sin\left(\underline{\omega}_0 \underline{t}_n - \underline{k}_0 \left|\underline{\mathbf{r}}_{i,j,k} - \underline{\mathbf{r}}_0\right|\right) \\ \underline{B}_z^{i,j,k,n} & = & 0 \end{cases}$$

with $\underline{E}_0 = a_f \underline{j}_0 / \underline{\omega}_0 = 10$, $\underline{\mathbf{r}}_0 = (64,\ 64)$ and $\underline{k}_0 = \underline{\omega}_0 / \underline{c} = 1$.

FTFD order 2 - Yee versus Spectrally Emulated

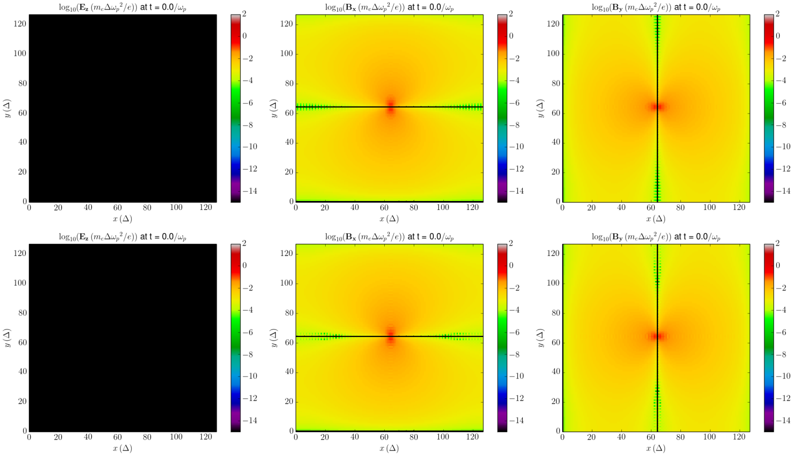


Figure 1: E_z (left), B_x (middle) and B_y (right) in log scale (top) and emulated (bottom)

Play/Pause

Emulated FTFD order $N = 128$ versus Spectral

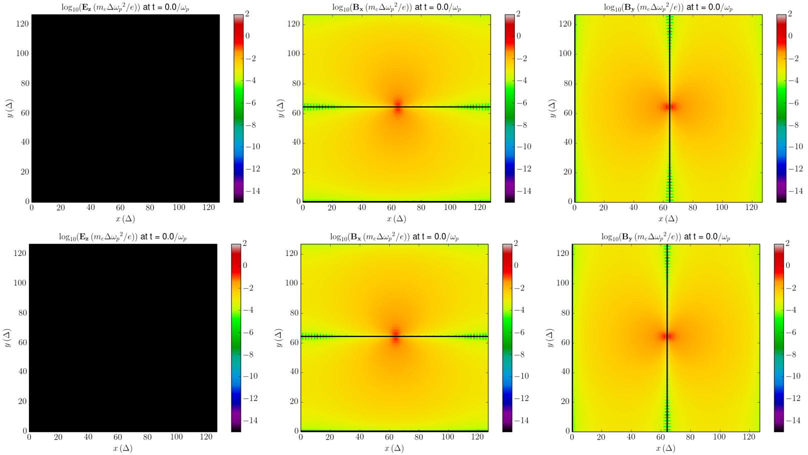
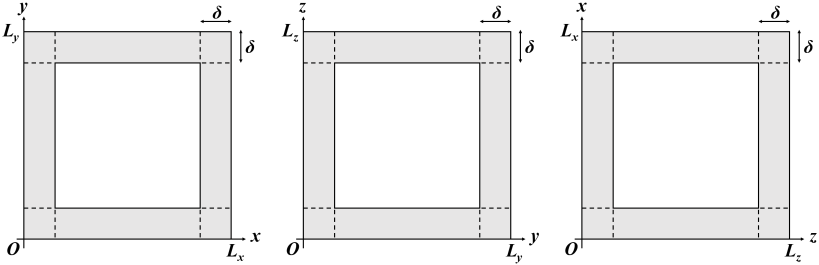


Figure 2: FTFD-128 E_z (left), B_x (middle) and B_y (right) in log scale (top) and spectral (bottom)

Play/Pause

Outline

Absorbing boundary conditions (1)



Michael D. Meyers Idea

The spectral solver remains exact if we dump the electromagnetic fields in the grey regions of thickness δ according to the Perfectly Matched Layers (PML) method developed for Finite Difference schemes such that the electromagnetic fields vanishes at the simulation box boundaries.

$$\begin{array}{llllll}
\forall j & \in [1,\ N_y]\,, & \forall k & \in [1,\ N_z]\,, \ \underline{\mathbf{E}}^{1,j,k,n} & = & \underline{\mathbf{E}}^{N_x,j,k,n} & = & \mathbf{0}, \\
\forall i & \in [1,\ N_x]\,, & \forall j & \in [1,\ N_y]\,, \ \underline{\mathbf{E}}^{i,j,1,n} & = & \underline{\mathbf{E}}^{i,j,N_z,n} & = & \mathbf{0} \text{ and} \\
\forall k & \in [1,\ N_z]\,, & \forall i & \in [1,\ N_x]\,, \ \underline{\mathbf{E}}^{i,1,k,n} & = & \underline{\mathbf{E}}^{i,N_y,k,n} & = & \mathbf{0}.
\end{array}$$

Absorbing boundary conditions (2)

PML method

1. Dumping terms are added into Maxwell-Ampere and Maxwell-Faraday :

$$\frac{\partial \mathbf{E}_T}{\partial \underline{t}} + \underline{\boldsymbol{\sigma}} \cdot \underline{\mathbf{E}}_T = \underline{c\nabla} \times \underline{\mathbf{B}} - a_f \underline{\mathbf{j}}_T \text{ and } \frac{\partial \underline{\mathbf{B}}}{\partial \underline{t}} + \underline{\boldsymbol{\sigma}}^* \cdot \underline{\mathbf{B}} = -\underline{c\nabla} \times \underline{\mathbf{E}}_T$$

2. Each component of the electromagnetic field is split into two parts :

$$\left\{\begin{array}{l} E_{T,x} = E_{xy} + E_{xz} \\ E_{T,y} = E_{yz} + E_{yx} \\ E_{T,z} = E_{zx} + E_{zy} \end{array}\right. \text{ and } \left\{\begin{array}{l} B_x = B_{xy} + B_{xz} \\ B_y = B_{yz} + B_{yx} \\ B_z = B_{zx} + B_{zy} \end{array}\right.$$

and we have to solve the PML equations

$$\left\{\begin{array}{l} \partial_{\underline{t}} E_{xy} + \underline{\sigma_y}(\underline{y})\, \underline{E_{xy}} = +\partial_{\underline{y}} \underline{B}_z^{\mathrm{PML}} \\ \partial_{\underline{t}} E_{xz} + \underline{\sigma_z}(\underline{z})\, \underline{E_{xz}} = -\partial_{\underline{z}} \underline{B}_y^{\mathrm{PML}} \\ \partial_{\underline{t}} E_{yz} + \underline{\sigma_z}(\underline{z})\, \underline{E_{yz}} = +\partial_{\underline{z}} \underline{B}_x^{\mathrm{PML}} \\ \partial_{\underline{t}} E_{yx} + \underline{\sigma_x}(\underline{x})\, \underline{E_{yx}} = -\partial_{\underline{x}} \underline{B}_z^{\mathrm{PML}} \\ \partial_{\underline{t}} E_{zx} + \underline{\sigma_x}(\underline{x})\, \underline{E_{zx}} = +\partial_{\underline{x}} \underline{B}_y^{\mathrm{PML}} \\ \partial_{\underline{t}} E_{zy} + \underline{\sigma_y}(\underline{y})\, \underline{E_{zy}} = -\partial_{\underline{y}} \underline{B}_x^{\mathrm{PML}} \end{array}\right. \text{ and } \left\{\begin{array}{l} \partial_{\underline{t}} B_{xy} + \underline{\sigma_y^*}(\underline{y})\, \underline{B_{xy}} = -\partial_{\underline{y}} \underline{E}_z^{\mathrm{PML}} \\ \partial_{\underline{t}} B_{xz} + \underline{\sigma_z^*}(\underline{z})\, \underline{B_{xz}} = +\partial_{\underline{z}} \underline{E}_y^{\mathrm{PML}} \\ \partial_{\underline{t}} B_{yz} + \underline{\sigma_z^*}(\underline{z})\, \underline{B_{yz}} = -\partial_{\underline{z}} \underline{E}_x^{\mathrm{PML}} \\ \partial_{\underline{t}} B_{yx} + \underline{\sigma_x^*}(\underline{x})\, \underline{B_{yx}} = +\partial_{\underline{x}} \underline{E}_z^{\mathrm{PML}} \\ \partial_{\underline{t}} B_{zx} + \underline{\sigma_x^*}(\underline{x})\, \underline{B_{zx}} = -\partial_{\underline{x}} \underline{E}_y^{\mathrm{PML}} \\ \partial_{\underline{t}} B_{zy} + \underline{\sigma_y^*}(\underline{y})\, \underline{B_{zy}} = +\partial_{\underline{y}} \underline{E}_x^{\mathrm{PML}} \end{array}\right.$$

with $\sigma_\xi^*(\xi) = \sigma_\xi(\xi) = \sigma_m(|\xi - \xi_I|/\delta)^n$ ($\xi = \xi_I \equiv$ PML medium / simulation box interface)

<u>Remark :</u> σ_ξ and σ_ξ^* depends on $\xi \Rightarrow$ The PML Equations are not linear!

Absorbing boundary conditions (3)

PML Equations - Yee scheme - Electric field update

$$
\left\{
\begin{aligned}
\underline{E}_{xy}^{i-1/2,j,k,n+1} &= \alpha_y^j \underline{E}_{xy}^{i-1/2,j,k,n} - \gamma_y^{i,j,k} j_{T,x}^{i-1/2,j,k,n+1/2}/2 \\
&+ \beta_y^j \sum_{m=1}^{s} C_m \left(\underline{B}_z^{i-1/2,j-1/2+m,k,n+1/2} - \underline{B}_z^{i-1/2,j-1/2-m+1,k,n+1/2} \right) \\
\underline{E}_{xz}^{i-1/2,j,k,n+1} &= \alpha_z^k \underline{E}_{xz}^{i-1/2,j,k,n} - \gamma_z^{i,j,k} j_{T,x}^{i-1/2,j,k,n+1/2}/2 \\
&- \beta_z^k \sum_{m=1}^{s} C_m \left(\underline{B}_y^{i-1/2,j,k-1/2+m,n+1/2} - \underline{B}_y^{i-1/2,j,k-1/2-m+1)/2,n+1/2} \right) \\
\underline{E}_{yz}^{i,j-1/2,k,n+1} &= \alpha_z^k \underline{E}_{yz}^{i,j-1/2,k,n} - \gamma_z^{i,j,k} j_{T,y}^{i-1/2,j,k,n+1/2}/2 \\
&+ \beta_z^k \sum_{m=1}^{s} C_m \left(\underline{B}_x^{i,j-1/2,k-1/2+m,n+1/2} - \underline{B}_x^{i,j-1/2,k-1/2-m+1,n+1/2} \right) \\
\underline{E}_{yx}^{i,j-1/2,k,n+1} &= \alpha_x^i \underline{E}_{yx}^{i,j-1/2,k,n} - \gamma_x^{i,j,k} j_{T,y}^{i-1/2,j,k,n+1/2}/2 \\
&- \beta_x^i \sum_{m=1}^{s} C_m \left(\underline{B}_z^{i-1/2+m,j-1/2,k,n+1/2} - \underline{B}_z^{i-1/2-m+1,j-1/2,k,n+1/2} \right) \\
\underline{E}_{zx}^{i,j,k-1/2,n+1} &= \alpha_x^i \underline{E}_{zx}^{i,j,k-1/2,n} - \gamma_x^{i,j,k} j_{T,z}^{i-1/2,j,k,n+1/2}/2 \\
&+ \beta_x^i \sum_{m=1}^{s} C_m \left(\underline{B}_y^{i-1/2+m,j,k-1/2,n+1/2} - \underline{B}_y^{i-1/2-m+1,j,k-1/2,n+1/2} \right) \\
\underline{E}_{zy}^{i,j,k-1/2,n+1} &= \alpha_y^j \underline{E}_{zy}^{i,j,k-1/2,n} - \gamma_y^{i,j,k} j_{T,z}^{i-1/2,j,k,n+1/2}/2 \\
&- \beta_y^j \sum_{m=1}^{s} C_m \left(\underline{B}_x^{i,j-1/2+m,k-1/2,n+1/2} - \underline{B}_x^{i,j-1/2-m+1,k-1/2,n+1/2} \right)
\end{aligned}
\right.
$$

with $\forall \xi \in \{x,\ y,\ z\}$, $\alpha_\xi^{i,j,k} = \dfrac{1 - \underline{\sigma}_\xi^{i,j,k} \underline{\Delta}_t/2}{1 + \underline{\sigma}_\xi^{i,j,k} \underline{\Delta}_t/2}$, $\beta_\xi^{i,j,k} = \dfrac{\underline{c}\,\underline{\Delta}_t/\underline{\Delta}_\xi}{1 + \sigma_\xi^{i,j,k}/2}$ and $\gamma_\xi^{i,j,k} = \dfrac{\underline{c}\,\underline{\Delta}_t}{1 + \sigma_\xi^{i,j,k}/2}$

Absorbing boundary conditions (4)

PML Equations - Yee scheme - Magnetic field update

$$
\left\{
\begin{aligned}
\underline{B}_{xy}^{i,j-1/2,k-1/2,n+1/2} &= \alpha_y^{*j-1/2}\underline{B}_{xy}^{i,j-1/2,k-1/2,n-1/2} \\
&\quad - \beta_y^{*j+1}\sum_{m=1}^{s} C_m\left(\underline{E}_z^{i,j+m-1,k-1/2,n} - \underline{E}_z^{i,j-m,k-1/2,n}\right) \\
\underline{B}_{xz}^{i,j-1/2,k-1/2,n+1/2} &= \alpha_z^{*k-1/2}\underline{B}_{xz}^{i,j-1/2,k-1/2,n-1/2} \\
&\quad + \beta_z^{*k-1/2}\sum_{m=1}^{s} C_m\left(\underline{E}_y^{i,j-1/2,k+m-1,n} - \underline{E}_y^{i,j-1/2,k-m,n}\right) \\
\underline{B}_{yz}^{i-1/2,j,k-1/2,n+1/2} &= \alpha_z^{*k-1/2}\underline{B}_{yz}^{i-1/2,j,k-1/2,n-1/2} \\
&\quad - \beta_z^{*k-1/2}\sum_{m=1}^{s} C_m\left(\underline{E}_x^{i-1/2,j,k+m-1,n} - \underline{E}_x^{i-1/2,j,k-m,n}\right) \\
\underline{B}_{yx}^{i-1/2,j,k-1/2,n+1/2} &= \alpha_x^{*i-1/2}\underline{B}_{yx}^{i-1/2,j,k-1/2,n-1/2} \\
&\quad + \beta_x^{*i-1/2}\sum_{m=1}^{s} C_m\left(\underline{E}_z^{i+m-1,j,k-1/2,n} - \underline{E}_z^{i-m,j,k-1/2,n}\right) \\
\underline{B}_{zx}^{i-1/2,j-1/2,k,n+1/2} &= \alpha_x^{*i-1/2}\underline{B}_{zx}^{i-1/2,j-1/2,k,n-1/2} \\
&\quad - \beta_x^{*i-1/2}\sum_{m=1}^{s} C_m\left(\underline{E}_y^{i+m-1,j-1/2,k,n} - \underline{E}_y^{i-m,j-1/2,k,n}\right) \\
\underline{B}_{zy}^{i-1/2,j-1/2,k,n+1/2} &= \alpha_y^{*j-1/2}\underline{B}_{zy}^{i-1/2,j-1/2,k,n-1/2} \\
&\quad + \beta_y^{*j-1/2}\sum_{m=1}^{s} C_m\left(\underline{E}_x^{i-1/2,j+m-1,k,n} - \underline{E}_x^{i-1/2,j-m,k,n}\right)
\end{aligned}
\right.
$$

$$
\text{with } \forall\xi \in \{x,\ y,\ z\},\ \alpha_\xi^{*i,j,k} = \frac{1-\underline{\sigma}_\xi^{*i,j,k}\underline{\Delta}_t/2}{1+\underline{\sigma}_\xi^{*i,j,k}\underline{\Delta}_t/2},\ \beta_\xi^{*i,j,k} = \frac{\underline{c}\,\underline{\Delta}_t/\underline{\Delta}_\xi}{1+\sigma_\xi^{*i,j,k}\underline{\Delta}_t/2}.
$$

Outline

Implementation in a Spectral Solver

Implementation in a Spectral Solver

0. $\underline{\widehat{E}}_{xy}^{p,q,r,0} = \underline{\widehat{E}}_{xz}^{p,q,r,0} = \underline{\widehat{E}}_{x}^{p,q,r,0}/2, \ \underline{\widehat{E}}_{yx}^{p,q,r,0} = \underline{\widehat{E}}_{yz}^{p,q,r,0} = \underline{\widehat{E}}_{y}^{p,q,r,0}/2$ etc...

1. $\underline{\widehat{\mathbf{B}}}^{p,q,r,n+1/2,*} = \underline{\widehat{\mathbf{B}}}^{p,q,r,n,*} - \frac{\underline{\Delta}_t}{2} \underline{c} \left(\iota \underline{\mathbf{k}}_{p,q,r}\right) \times \underline{\widehat{\mathbf{E}}}_T^{p,q,r,n}$

2. $$\begin{cases} \underline{\widehat{B}}_{\xi\zeta}^{p,q,r,n+1/2} = \text{IDFT}\left\{ \frac{1}{1+\underline{\sigma}_\zeta^{i,j,k}\underline{\Delta}_t/2} \text{DFT}\left\{\underline{\widehat{B}}_{\xi\zeta}^{p,q,r,n+1/2,*}\right\}^{i,j,k}\right\}^{p,q,r} \\ \underline{\widehat{E}}_{\xi\zeta}^{p,q,r,n,*} = \text{IDFT}\left\{\left(1-\underline{\sigma}_\zeta^{i,j,k}\underline{\Delta}_t/2\right) \text{DFT}\left\{\underline{\widehat{E}}_{\xi\zeta}^{p,q,r,n}\right\}^{i,j,k}\right\}^{p,q,r} \end{cases}$$

3. $\underline{\widehat{\mathbf{E}}}_T^{p,q,r,n+1,*} = \underline{\widehat{\mathbf{E}}}_T^{p,q,r,n,*} + \underline{\Delta}_t \underline{c} \left(\iota \underline{\mathbf{k}}_{p,q,r}\right) \times \underline{\widehat{\mathbf{B}}}^{p,q,r,n+1/2} - \underline{\Delta}_t \underline{a}_f \underline{\widehat{\mathbf{j}}}_T^{p,q,r,n+1/2}$

4. $$\begin{cases} \underline{\widehat{E}}_{\xi\zeta}^{p,q,r,n+1} = \text{IDFT}\left\{ \frac{1}{1+\underline{\sigma}_\zeta^{i,j,k}\underline{\Delta}_t/2} \text{DFT}\left\{\underline{\widehat{E}}_{\xi\zeta}^{p,q,r,n+1,*}\right\}^{i,j,k}\right\}^{p,q,r} \\ \underline{\widehat{B}}_{\xi\zeta}^{p,q,r,n+1/2,*} = \text{IDFT}\left\{\left(1-\underline{\sigma}_\zeta^{i,j,k}\underline{\Delta}_t/2\right) \text{DFT}\left\{\underline{\widehat{B}}_{\xi\zeta}^{p,q,r,n-1/2}\right\}^{i,j,k}\right\}^{p,q,r} \end{cases}$$

5. $\underline{\widehat{\mathbf{B}}}^{p,q,r,n+1,*} = \underline{\widehat{\mathbf{B}}}^{p,q,r,n+1/2,*} - \frac{\underline{\Delta}_t}{2} \underline{c} \left(\iota \underline{\mathbf{k}}_{p,q,r}\right) \times \underline{\widehat{\mathbf{E}}}_T^{p,q,r,n+1}$

6. $$\underline{\widehat{B}}_{\xi\zeta}^{p,q,r,n+1} = \text{IDFT}\left\{ \frac{1}{1+\underline{\sigma}_\zeta^{i,j,k}\underline{\Delta}_t/2} \text{DFT}\left\{\underline{\widehat{B}}_{\xi\zeta}^{p,q,r,n+1,*}\right\}^{i,j,k}\right\}^{p,q,r}$$

7. $\underline{\widehat{E}}_{x}^{p,q,r,n+1} = \underline{\widehat{E}}_{xz}^{p,q,r,n+1} + \underline{\widehat{E}}_{xy}^{p,q,r,n+1}, \ \underline{\widehat{E}}_{y}^{p,q,r,n+1} = \underline{\widehat{E}}_{yx}^{p,q,r,n+1} + \underline{\widehat{E}}_{yz}^{p,q,r,n+1}$ etc...

Steps 1, 3 and 5 actually concerns the splitted PML components $\underline{\widehat{E}}_{xy}^{p,q,r,n}$, $\underline{\widehat{E}}_{xz}^{p,q,r,0}$ etc... and NOT the common fields!
It has been written like this just for illustrating the scheme more efficiently.

Outline

Fully Implicit Scheme for the Dumping Terms (1)

PML Equations - Yee scheme except for the dumping term taken fully implicitely - Electric field update

$$
\left\{
\begin{array}{lcl}
E_{xy}^{i-1/2,j,k,n+1} & = & \alpha_y^j E_{xy}^{i-1/2,j,k,n} - \gamma_y^{i,j,k} j_{T,x}^{i-1/2,j,k,n+1/2}/2 \\
 & + & \beta_y^j \sum_{m=1}^{s} C_m \left(B_z^{i-1/2,j-1/2+m,k,n+1/2} - B_z^{i-1/2,j-1/2-m+1,k,n+1/2} \right) \\
E_{xz}^{i-1/2,j,k,n+1} & = & \alpha_z^k E_{xz}^{i-1/2,j,k,n} - \gamma_z^{i,j,k} j_{T,x}^{i-1/2,j,k,n+1/2}/2 \\
 & - & \beta_z^k \sum_{m=1}^{s} C_m \left(B_y^{i-1/2,j,k-1/2+m,n+1/2} - B_y^{i-1/2,j,k-1/2-m+1)/2,n+1/2} \right) \\
E_{yz}^{i,j-1/2,k,n+1} & = & \alpha_z^k E_{yz}^{i,j-1/2,k,n} - \gamma_z^{i,j,k} j_{T,y}^{i-1/2,j,k,n+1/2}/2 \\
 & + & \beta_z^k \sum_{m=1}^{s} C_m \left(B_x^{i,j-1/2,k-1/2+m,n+1/2} - B_x^{i,j-1/2,k-1/2-m+1,n+1/2} \right) \\
E_{yx}^{i,j-1/2,k,n+1} & = & \alpha_x^i E_{yx}^{i,j-1/2,k,n} - \gamma_x^{i,j,k} j_{T,y}^{i-1/2,j,k,n+1/2}/2 \\
 & - & \beta_x^i \sum_{m=1}^{s} C_m \left(B_z^{i-1/2+m,j-1/2,k,n+1/2} - B_z^{i-1/2-m+1,j-1/2,k,n+1/2} \right) \\
E_{zx}^{i,j,k-1/2,n+1} & = & \alpha_x^i E_{zx}^{i,j,k-1/2,n} - \gamma_x^{i,j,k} j_{T,z}^{i-1/2,j,k,n+1/2}/2 \\
 & + & \beta_x^i \sum_{m=1}^{s} C_m \left(B_y^{i-1/2+m,j,k-1/2,n+1/2} - B_y^{i-1/2-m+1,j,k-1/2,n+1/2} \right) \\
E_{zy}^{i,j,k-1/2,n+1} & = & \alpha_y^j E_{zy}^{i,j,k-1/2,n} - \gamma_y^{i,j,k} j_{T,z}^{i-1/2,j,k,n+1/2}/2 \\
 & - & \beta_y^j \sum_{m=1}^{s} C_m \left(B_x^{i,j-1/2+m,k-1/2,n+1/2} - B_x^{i,j-1/2-m+1,k-1/2,n+1/2} \right)
\end{array}
\right.
$$

with $\forall \xi \in \{x,\ y,\ z\}$, $\alpha_\xi^{i,j,k} = \dfrac{1}{1+\sigma_\xi^{i,j,k}\Delta_t/2}$, $\beta_\xi^{i,j,k} = \dfrac{c\,\Delta_t/\Delta_\xi}{1+\sigma_\xi^{i,j,k}/2}$ and $\gamma_\xi^{i,j,k} = \dfrac{c\,\Delta_t}{1+\sigma_\xi^{i,j,k}/2}$

Fully Implicit Scheme for the Dumping Terms (2)

Implicit Scheme

0. $\underline{\widehat{E}}_{xy}^{p,q,r,0} = \underline{\widehat{E}}_{xz}^{p,q,r,0} = \underline{\widehat{E}}_{x}^{p,q,r,0}/2,\ \underline{\widehat{E}}_{yx}^{p,q,r,0} = \underline{\widehat{E}}_{yz}^{p,q,r,0} = \underline{\widehat{E}}_{y}^{p,q,r,0}/2$ etc...
1. $\underline{\widehat{\mathbf{B}}}^{p,q,r,n+1/2,*} = \underline{\widehat{\mathbf{B}}}^{p,q,r,n,*} - \frac{\Delta_t}{2}\underline{c}\left(\iota \underline{\mathbf{k}}_{p,q,r}\right) \times \underline{\widehat{\mathbf{E}}}_T^{p,q,r,n}$
2. $\underline{\widehat{B}}_{\xi\zeta}^{p,q,r,n+1/2} = \mathrm{IDFT}\left\{\frac{1}{1+\underline{\sigma}_{\zeta}^{i,j,k}\Delta_t/2}\ \mathrm{DFT}\left\{\underline{\widehat{B}}_{\xi\zeta}^{p,q,r,n+1/2,*}\right\}^{i,j,k}\right\}^{p,q,r}$
3. $\underline{\widehat{E}}_T^{p,q,r,n+1,*} = \underline{\widehat{\mathbf{E}}}_T^{p,q,r,n,*} + \underline{\Delta}_t \underline{c}\left(\iota \underline{\mathbf{k}}_{p,q,r}\right) \times \underline{\widehat{\mathbf{B}}}^{p,q,r,n+1/2} - \underline{\Delta}_t a_f\, \underline{\widehat{\mathbf{j}}}_T^{p,q,r,n+1/2}$
4. $\underline{\widehat{E}}_{\xi\zeta}^{p,q,r,n+1} = \mathrm{IDFT}\left\{\frac{1}{1+\underline{\sigma}_{\zeta}^{i,j,k}\Delta_t/2}\ \mathrm{DFT}\left\{\underline{\widehat{E}}_{\xi\zeta}^{p,q,r,n+1,*}\right\}^{i,j,k}\right\}^{p,q,r}$
5. $\underline{\widehat{\mathbf{B}}}^{p,q,r,n+1,*} = \underline{\widehat{\mathbf{B}}}^{p,q,r,n+1/2,*} - \frac{\Delta_t}{2}\underline{c}\left(\iota \underline{\mathbf{k}}_{p,q,r}\right) \times \underline{\widehat{\mathbf{E}}}_T^{p,q,r,n+1}$
6. $\underline{\widehat{B}}_{\xi\zeta}^{p,q,r,n+1} = \mathrm{IDFT}\left\{\frac{1}{1+\underline{\sigma}_{\zeta}^{i,j,k}\Delta_t/2}\mathrm{DFT}\left\{\underline{\widehat{B}}_{\xi\zeta}^{p,q,r,n+1,*}\right\}^{i,j,k}\right\}^{p,q,r}$
7. $\underline{\widehat{E}}_{x}^{p,q,r,n+1} = \underline{\widehat{E}}_{xz}^{p,q,r,n+1} + \underline{\widehat{E}}_{xy}^{p,q,r,n+1},\ \underline{\widehat{E}}_{y}^{p,q,r,n+1} = \underline{\widehat{E}}_{yx}^{p,q,r,n+1} + \underline{\widehat{E}}_{yz}^{p,q,r,n+1}$ etc...

Steps 1, 3 and 5 actually concerns the splitted PML components $\underline{\widehat{E}}_{xy}^{p,q,r,n}$, $\underline{\widehat{E}}_{xz}^{p,q,r,0}$ etc... and NOT the common fields!
It has been written like this just for illustrating the scheme more efficiently.

Initialization

Nominal Simulation Parameters

- 2D
- Filter used with $\underline{a}_x = \underline{a}_y = .912871$
- Electromagnetic units ($\underline{c} = 1$)
- 4 MPI nodes and 2 OpenMP threads per node
- $\underline{L}_x = \underline{L}_y = 128$
- $\underline{\delta} = 30$
- $\underline{\Delta}_x = \underline{\Delta}_y = 1$
- $\underline{L}_t = 128$
- $a_f = 1$

Best PML conductivity expression for the FTFD Yee Scheme [1]

$$\begin{cases} \underline{\sigma}_x(\underline{x}) = \underline{\sigma}_m \left(\frac{|\underline{x} - \underline{x}_l|}{\underline{\delta}} \right)^\beta \\ \underline{\sigma}_y(\underline{y}) = \underline{\sigma}_m \left(\frac{|\underline{y} - \underline{y}_l|}{\underline{\delta}} \right)^\beta \end{cases} \quad \text{where}$$

$$3 \leq \beta \leq 4 \text{ and } \underline{\sigma}_m = \frac{\alpha \ln 10 \, (\beta + 1)\, \underline{c}}{2\underline{\delta}} = 0.8\,(\beta + 1)\,\underline{c}$$

[1] Computational electrodynamics, the FDTD method, A. Taflove and S C Hagness, Eq. (7.62) and (7.66)

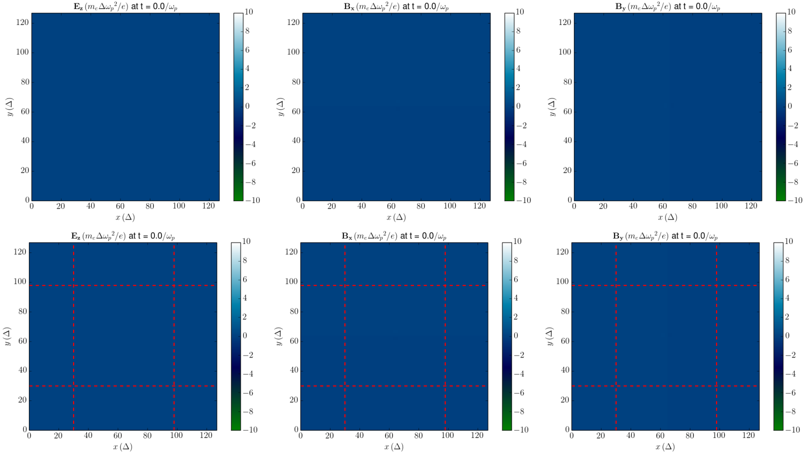
Electrical Current Density Initialization and Expected Solutions

$$\underline{\mathbf{j}}_T^{i,j,k,n} = \begin{cases} \underline{j}_0 \sin(\underline{\omega}_0 \underline{t}_n)\mathbf{e}_z & \text{if } \begin{cases} 63 \leq \underline{x}_i \leq 65 \\ 63 \leq \underline{y}_j \leq 65 \end{cases} \\ 0 & \text{else} \end{cases} \quad \text{with } \underline{\omega}_0 = 1 \text{ and } \underline{j}_0 = -10.$$

$$\Rightarrow \begin{cases} \underline{E}_{T,r}^{i,j,k,n} = 0 \\ \underline{E}_{T,z}^{i,j,k,n} \underset{\underline{k}_0 \left|\underline{\mathbf{r}}_{i,j,k} - \underline{\mathbf{r}}_0\right| \gg 1}{\approx} \frac{\underline{E}_0}{\sqrt{\left|\underline{\mathbf{r}}_{i,j,k} - \underline{\mathbf{r}}_0\right|}} \cos\left(\underline{\omega}_0 \underline{t}_n - \underline{k}_0 \left|\underline{\mathbf{r}}_{i,j,k} - \underline{\mathbf{r}}_0\right|\right) \\ \underline{B}_r^{i,j,k,n} \underset{\underline{k}_0 \left|\underline{\mathbf{r}}_{i,j,k} - \underline{\mathbf{r}}_0\right| \gg 1}{\approx} \frac{\underline{E}_0}{\sqrt{\left|\underline{\mathbf{r}}_{i,j,k} - \underline{\mathbf{r}}_0\right|}} \sin\left(\underline{\omega}_0 \underline{t}_n - \underline{k}_0 \left|\underline{\mathbf{r}}_{i,j,k} - \underline{\mathbf{r}}_0\right|\right) \\ \underline{B}_z^{i,j,k,n} = 0 \end{cases}$$

with $\underline{E}_0 = a_f \underline{j}_0 / \underline{\omega}_0 = 10$, $\underline{\mathbf{r}}_0 = (64,\ 64)$ and $\underline{k}_0 = \underline{\omega}_0 / \underline{c} = 1$.

Periodic versus Absorbing



E_z (left), B_x (middle) and B_y (right) with periodic (top) and PML-Yee (bottom) boundary conditions

Play/Pause